Empirical Bayes Credibility Models for Economic Catastrophic Losses by Regions

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Abstract. Catastrophic events affect various regions of the world with increasing frequency and intensity. The number of catastrophic events and the amount of economic losses is varying in different regions of the world. Part of these losses is covered by insurance. Catastrophe events in last years are associated with increases in premiums for some lines of business. The article focus on estimating the amount of net premiums that would be needed to cover the total or insured catastrophic losses in different regions of the world using empirical credibility models.

1 Introduction

Based on *Sigma Swiss Re* terminology [19] the term *natural catastrophe* refers to an event caused by natural forces. Such an event generally results in a large number of individual losses involving many insurance policies. An event is classified as a catastrophe and included in the *sigma* database when insured claims, total losses or the number of casualties exceed certain thresholds, detailed in Table1.

Table 1. The sigma event selection criteria 2015

Insured losses:	
Maritime disasters	19.7 USD million
Aviation	39.3 USD million
Other losses	48.8 USD million
or Total economic losses:	97.7 USD million
or Casualties:	
Dead or missing	20
Injured	50
Homeless	2000

Source: Sigma Swiss Re, No 1/2016, [19].

The occurrences of catastrophic events are becoming more frequent (Fig. 1) and also grow indemnity of insurance and reinsurance companies at these events (Fig. 2).
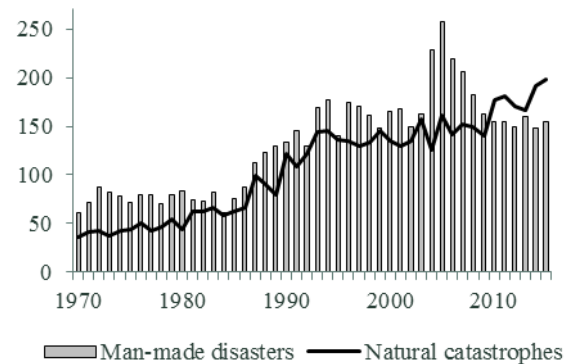
Source: Sigma Swiss Re, No 1/2016, [19].
**Fig. 1.** Number of catastrophes events 1970-2015

By *sigma* [19] there were 198 natural catastrophes in 2015, and 155 man-made disaster events. The estimated total economic losses from natural catastrophes and man-made disasters across the world were USD 92 billion in 2015. This is less than the USD 113 billion total loss in 2014, and is well below the inflation-adjusted average of the previous 10 years (USD 192 billion).

The insurance industry covered almost USD 37 billion – less than half – of the total losses from natural and man-made disasters in 2015, and well below the inflationadjusted previous 10-year annual average of USD 62 billion. Natural catastrophes resulted in claims of USD 28 billion, the lowest since 2009 and again much lower than the previous 10-year inflation-adjusted annual average (USD 55 billion). Large man-made disasters led

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to claims of USD 9 billion, up from USD 7 billion in 2014.

Fig. 2 shows also the difference between insured and total losses over time period 1970-2015, termed the insurance protection or funding gap. It is the amount of financial loss generated by catastrophes not covered by insurance. In this figure it is evident growing trend both total and insured catastrophe losses in time period 1970-2015, determined by 10-years moving averages. The rate of growth of total losses has outpaced the growth of insured losses.
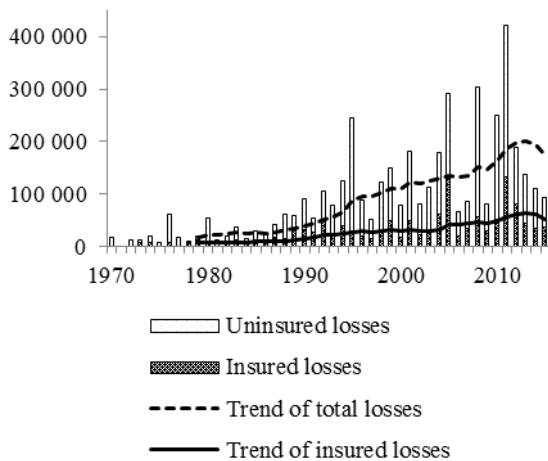

Source: Sigma Swiss Re, No 1/2016
Fig. 2. Insured and uninsured losses in 1970-2015 in USD million in 2015 prices

Catastrophic events affect various regions of the world with increasing frequency and intensity. Large catastrophic events can be caused by natural phenomena or are caused by man.

The number of catastrophic events is also varying in different regions of the world. The catastrophic events by Sigma Swiss Re are monitored for seven regions - North America, Latin America, Europe, Africa, Asia, Oceania/Australia and Seas/space. Number of catastrophe events by regions in 2009-2015 present Table 2 and Fig. 3.
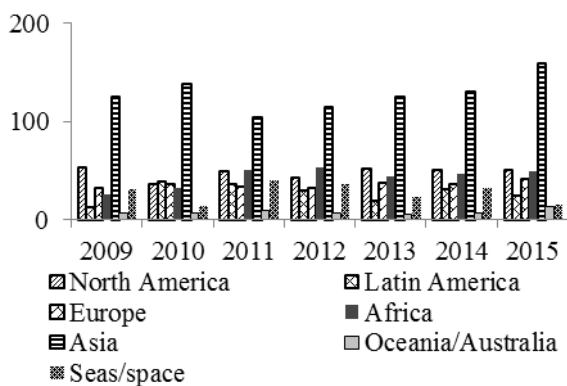
Source: Own processing by Sigma Swiss Re, 2010-2016
Fig. 3. Number of catastrophe events by regions for 2009-2015

The worldwide insurance industry has been rocked by the increasing catastrophes in recent years and increased demand for catastrophe cover (e.g., per occurrence excess of loss reinsurance), leading to a capacity shortage in catastrophe reinsurance. Catastrophe events in last years are associated with increases in premiums for some lines of business.

Table 2. Number of catastrophe events by regions (2009-2015)

	2009	2010	2011	2012	2013	2014	2015
North America	54	36	50	43	52	51	51
Latin America	13	39	36	30	20	31	25
Europe	32	37	34	33	38	37	41
Africa	26	32	51	53	44	47	49
Asia	125	139	104	115	125	130	159
Oceania/ Australia	7	7	10	7	6	7	13
Seas / space	31	14	40	37	23	33	15

Source: Own processing by Sigma Swiss Re, 2010-2016

Regional differences can be observed not only in the number of catastrophic events, but also in the amount of economic losses. Part of these losses is covered by insurance. The share of insured losses on total losses again varies by region.

Table 3. Total economic losses in USD bn.

	2009	2010	2011	2012	2013	2014	2015
North America	20.1	20.6	63.5	118.5	32.0	28.6	28.6
Latin America	0.6	53.4	5.6	4.2	9.0	8.2	7.5
Europe	20.1	35.2	8.7	26.8	33.0	15.9	12.6
Africa	0.5	0.3	1.6	1.5	1.0	1.5	1.2
Asia	16.7	78.8	260.1	30.5	62.0	51.7	37.7
Oceania/ Australia	2.1	13.1	27.8	1.1	3.0	2.3	3.0
Seas / space	2.0	20.6	3.6	3.1	1.0	1.7	1.1

Source: Own processing by Sigma Swiss Re, 2010-2016

The collected premiums used to cover the insured losses not only in the region of insured, but if necessary,

in other regions of the world. It is caused by the globalization of the insurance and reinsurance market.

Table 4. Insurance losses by region in 2009-2015 in USD billion

	2009	2010	2011	2012	2013	2014	2015
North America	12.7	15.3	39.8	64.6	19.0	17.5	17.3
Latin America	0.1	9.0	0.6	0.9	2.0	2.3	3.2
Europe	7.7	6.3	4.3	5.5	15.0	6.6	6.2
Africa	0.2	0.1	0.3	0.2	1.0	0.8	0.0
Asia	2.4	2.2	49.2	3.4	6.0	5.2	7.0
Oceania/ Australia	1.3	8.9	19.1	0.3	1.0	1.0	2.1
Seas / space	2.0	1.6	2.4	2.4	1.0	1.3	0.9

Source: Own processing by Sigma Swiss Re, 2010-2016

The article will therefore focus on estimating the amount of net premiums that would be needed to cover the total catastrophic losses in each region. For this purpose we will use empirical credibility models.

2 Application of Bühlmann credibility model

Credibility theory is a technique, or set of techniques, for calculating premiums for short term insurance contracts. The technique calculates a premium for a risk (in our case for a region) using two ingredients: past data from the risk itself and collateral data, i.e. data from other sources considered to be relevant, in our cases from other regions. The essential features of a credibility premium are that it is a linear function of the past data from the risk itself and that it allows for the premium to be regularly updated as more data are collected in the future [1, 2, 12, 13, 15, 18].

A credibility premium represents a compromise between the two above mentioned sources of information. The credibility formula for estimation of pure premium Pc in next year is [6, 7]:

$$P_c = Z\, P_r + (1 - Z)\, \mu \qquad (1)$$

where Pr is estimation based on own past data in risk, or region, μ is estimation based on collateral data and Z is a number between zero and one, known as the *credibility factor*. Credibility factor Z is a measure of how much reliance the company is prepared to place on the own data [8, 9, 10].

Empirical Bayes credibility theory is the collective name for the vast literature which has developed since Bühlmann [2] and Bühlmann and Straub [3].

The form of empirical Bayes credibility premium as a result of Bühlmann credibility model has been derived in [2] as

$$E\big(m(\theta)/\boldsymbol{x}\big) = Z\,\overline{x} + (1 - Z)E\big(m(\theta)\big) \qquad (2)$$

with factor credibility

$$Z = \cfrac{n}{n + \cfrac{E\big(s^2(\theta)\big)}{D\big(m(\theta)\big)}} \qquad (3)$$

In empirical Bayes credibility model we use the available data (Tab. 3) to estimate the quantities $E\big(m(\theta)\big)$, $D\big(m(\theta)\big)$, $E\big(s^2(\theta)\big)$, and hence obtain a Bühlman type credibility premium estimate [2] for a particular risk.
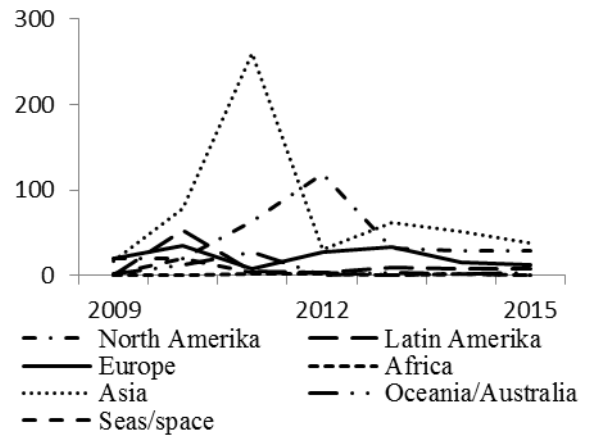
Derived relations necessary for estimates are:

$$est\, E\big(m(\theta)\big) = \overline{X} \qquad (4)$$

$$est\,E\big(s^2(\theta)\big) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{n-1}\sum_{j=1}^{n}\big(X_{ij} - \overline{X}_i\big)^2 \qquad (5)$$

$$est\,D\big(m(\theta)\big) = \begin{aligned} &\frac{1}{N-1}\sum_{i=1}^{N}\big(\overline{X}_i - \overline{X}\big)^2 - \\ &-\frac{1}{Nn}\sum_{i=1}^{N}\frac{1}{n-1}\sum_{j=1}^{n}\big(X_{ij} - \overline{X}_i\big)^2 \end{aligned} \qquad (6)$$

The derivation of these relations can be found in Bühlmann [2] and Waters [18].

Table 3 and Fig. 4 give present the total economic losses (in USD billion) to each of seven regions of the world over seven years.

Source: Own processing by Sigma Swiss Re, 2010-2016
Fig. 4. Times series plot of total economic losses in USD bn

Using the Bühlmann credibility model we have calculated the credibility factor and credibility premium for next year for each of the seven regions. For each of the regions we calculate means $\overline{X}_i = \sum_{j=1}^{n} X_{ij}/n$. These values contains second column in Table 5.

Table 5. Results of Bühlmann credibility model

	$\overline{X}_i$	Credibility premium
North America	44.54	39.66
Latin America	12.63	15.38
Europe	21.76	22.33
Africa	1.08	6.60
Asia	76.80	64.21
Oceania / Australia	7.48	11.47
Seas/space	4.73	9.38

Source: Own calculations

Using expressions (4) - (6) we have estimated $E(m(\theta))$, $E(s^2(\theta))$, $D(m(\theta))$:

$$estE(m(\theta)) = 24.1481;$$
$$estE(s^2(\theta)) = 1259.777;$$
$$estD(m(\theta)) = 572.2954$$

Therefore using (3) we have calculated the credibility factor Z which is the same for each region:

$$Z = 0{,}7608$$

The credibility premium for each of seven regions in USD billion by (2) is in last column of Table 6.

3 Application of Bühlmann - Straub credibility model

Contrary to the Bühlmann model in section 3 Bühlmann-Straub credibility model accounts for the varying annual volume of business, as is for example observed number of claims. We have applied this model on insured catastrophe losses (Table 4) and figure 5 with acceptation of the number of catastrophe events (Table 2).

Let region, for which the estimates of insurance catastrophic loses are computed, is one of N regions. Insured losses in previous n years are known for all these regions.

Let P_{ij} is a variable, describing number of catastrophic events of the i-th observed region ($i = 1,…, N$; $N = 7$) in the j-th year ($j = 1, …, n$; $n = 7$).

Let Y_{ij} is the size of insured catastrophe losses for each region $i = 1, …, N$ and each year $j = 1, …, n$.
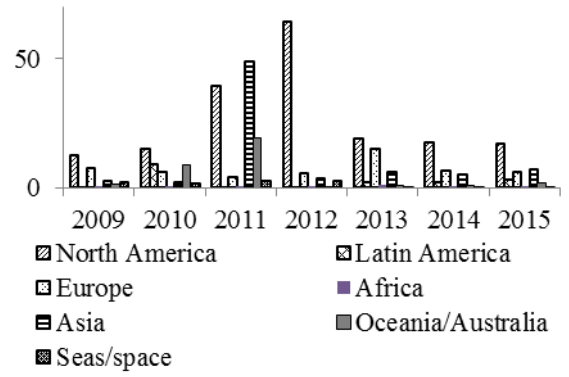
Fig. 5. Insured catastrophe losses by regions in 2009-2015 in USD billion

According to [4, 5, 11, 13, 16, 17, 18] for the estimation of credible net insured premium for the i-th risk, the following formula is used

$$E(m(\theta)/X) = Z_i\overline{X}_i + (1-Z_i)E(m(\theta))$$
$$= Z_i\overline{X}_i + (1-Z_i)\overline{X} \tag{7}$$

Credibility factor for the i-th risk is calculated in form [3, 18]:

$$Z_i = \frac{P_i}{P_i + \dfrac{E(s^2(\theta))}{D(m(\theta))}} \tag{8}$$

The rules for estimating the parameters $E(m(\theta))$, $E(s^2(\theta))$, $D(m(\theta))$ are by [3, 18]:

$$estE(m(\theta)) = \overline{X} \tag{9}$$

$$estE(s^2(\theta)) = \frac{1}{N(n-1)}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}\left(X_{ij} - \overline{X}_i\right)^2 \tag{10}$$

$$estD(m(\theta)) = \frac{1}{P^*}\left\{\frac{1}{Nn-1}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}\left(X_{ij} - \overline{X}\right)^2 - \frac{1}{N(n-1)}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}\left(X_{ij} - \overline{X}_i\right)^2\right\} \tag{11}$$

when it is mean, that

$$\overline{X}_i = \frac{1}{P_i}\sum_{j=1}^{n}P_{ij}X_{ij} = \frac{1}{P_i}\sum_{i=1}^{n}Y_{ij} \tag{12}$$

$$\overline{X} = \frac{1}{P}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}X_{ij} = \frac{1}{P}\sum_{i=1}^{N}P_i\overline{X}_i \tag{13}$$

$$P^* = \frac{1}{Nn-1}\sum_{i=1}^{N}P_i\cdot\left(1-\frac{P_i}{P}\right) \tag{14}$$

The value of credibility factor shows the rate of reliability of own data for each region.

The estimates of parameters $E(m(\theta))$, $D(m(\theta))$, $E(s^2(\theta))$ are the same for all the regions. Credibility factor Z_i differs for each region. The higher is the value of risk rate P_i, the higher is the value of credibility factor Z_i.

Bühlmann-Straub model was applied to compute credible net premium for seven regions of the world based insured catastrophe losses in time period 2009-2015 published in Sigma Swiss Re [20] (Table 4).

Total amount of insured losses Y_i, where $Y_i = \sum_{j=1}^{n} Y_{ij}$, insured by each region i in the time 2009-2015, total number of catastrophic events P_i, where $P_i = \sum_{j=1}^{n} P_{ij}$ by each region in the whole period, and average insured catastrophe losses per insurance events $\overline{X}_i$ for each region were computed according to (12). These characteristics present Table 6.

Table 6. Computed characteristics of the regions

	P_i	Y_i	$\overline{X}_i$
North America	337	186.16	0.5524
Latin America	194	18.06	0.0931
Europe	252	51.64	0.2049
Africa	302	2.67	0.0088
Asia	897	75.53	0.0842
Oceania / Australia	57	33.66	0.5906
Seas/space	193	11.59	0.0600

Source: Own calculations

Computed values in Table 6 have used to estimate parameters of the Bühlmann-Straub credibility model according to (9)-(11):

$$estE(m(\theta)) = 0.1699,$$
$$estE(s^2(\theta)) = 2.4757,$$
$$estD(m(\theta)) = 0.0366$$

According to (8), values of credibility factors Z_i are computed for each region. It is necessary to find these values to be able to estimate credible net premium per region according to (7). Values of credibility factors and values of credible net premiums by regions contain Tab. 7.

Table 7. Credibility factor Z_i and estimates of net insurance premium per region in 2016.

	Z_i	Net insurance premium
North America	0.8327	0.4884
Latin America	0.7413	0.1130
Europe	0.7882	0.1975
Africa	0.8168	0.0383
Asia	0.9298	0.0902
Oceania / Australia	0.4570	0.3622
Seas/space	0.7403	0.0886

Source: Own calculations

Estimates of credible net premiums by regions provide information how much money in the region will be needed to cover insured catastrophe losses in next year.

4 Conclusion

Developments of the financial consequences of natural and man-made catastrophes have a major impact on the global insurance market and forcing the insurance and reinsurance companies to seek for new approaches and ways to cover these risks [14]. Raises the concern that the capacity of the world's insurance and reinsurance markets in the future will not be sufficient to cover these risks and aims to seek alternative options for their transfer.

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