PARAMETERS OF DISPERSION FOR ON-TIME PERFORMANCE OF POSTAL ITEMS WITHIN TRANSIT TIMES MEASUREMENT SYSTEM FOR POSTAL SERVICES

Daniel Salava – Kateřina Pojkarová, Libor Švadlenka

Abstract

The paper is focused on usage of methods for on-time performance evaluation of test postal items flows used for measurement of the transit time of end-to-end postal services for single piece priority mail. The results come out from modelling of measurement system for postal transport network efficiency in proportion to real postal traffic flows with particular discriminant characteristics. The result of on-time performance is represented by estimate of on-time probability, which can be defined as probability of case, when transit time of postal item does not exceed just one day. On-time probability is interpreted by variable called weighted estimate of on-time probability, which also includes distribution of mail flows within geographical stratification according to disjunctive set of defined postal areas. The rate of variability caused by used sample design is expressed by parameters of sample dispersion in various forms and by design factors regarding particular samples. Assessment of these calculated key parameters is the main objective of this paper, because they evaluate measurement system efficiency and could serve as base for next modelling of this system. Calculation of these parameters presumes fixed postal transportation network and domestic mail flows for various sample sizes.

Key words: transit time, priority mail, sample dispersion, measurement system

JEL Code: C02, C13

Introduction

This paper is focused on usage of methods for assessment of test postal items flows used for measurement of the transit time of end-to-end transportation services for single piece priority mail (SPPM). The results come out from modelling of measurement of postal transport system efficiency on the basis of real postal traffic flows with particular discriminant characteristics. Measurement of transit time is realized by flow of test postal items in proportion to real postal traffic flows.

Because measuring of transit times is realized by representative sample of test letters, the result of on-time performance is represented by estimate of on-time probability. On-time probability for test letters can be defined as probability of case, when transit time of test letter flow does not exceed just one day. On-time probability is interpreted by variable called value of estimate for on-time probability, casely weighted estimate of on-time probability including distribution of letter flows within geographical stratification according to disjunctive set of defined postal areas. The rate of variation caused by used sample design is expressed by design factor from possible viewpoint of unstratified end-to-end measuring system, stratified random sample and stratified end-to-end sample (ČSN EN 13850, 2013). It is related to sample design and on-time probability estimate. Significant parameters of dispersion describing exploitation of measuring system are represented by sample variance.

Dispersion has very significant role to reflect variability features of samples or measuring data. It can be expressed by various tools or parameters like standard deviation, sample variance and so on. They generally have own constructions and possibilities of usage. Dispersion measuring also may be interpreted by various terms such variability, scale, spread and so on. Last mentioned spread can include more general point of view (Wilcox, 2005). Dispersion can have narrower conception of variability related to given fix point (Bickel and Lehmann, 1976). Relation of sample variance and mean value can serve as example.

Some known measures of dispersion consider one-dimensional data, but it has not to be sufficient approach for contemporary needs of data processing and assessment. Analysis of multidimensional data naturally gives possibility to measure dispersion separately for each variable, but possible relations between variables remain urevealed. Possible way to express that is covariance matrix with variances of all single variables and covariances for all pairs of variables. This way which leads to multidimensional sample with researches of some dimension with matrix of numbers equals square of this dimension (Kolacz and Grzegorzewski, 2016).

Dispersion measuring can apply also the aggregation theory with other reflection of spread measuring with one-dimensional data (Gagolewski, 2015). It is inspiring, because this theory is focused particularly on measures of central tendency (Beliakov, Pradera and Calvo, 2007), also known as measures of location or centrality of observations – sample quantiles as median, max ,min, and then mode, arithmetic mean and so on. This theory is strongly developed (Calvo and Beliakov, 2010) and definition of aggregation function has wide equipment of specific means (Grabisch, Marichal, Mesiar and Pap, 2011). For instance, the sample standard deviation can be applied to evaluate coherence of managerial decision making (Huang, Chang and Lin, 2013). There exist some other studies on the extended spread measures (Calvo and Mayor, 1999), studies connected with multi-argument distances or relationship between multidimensional dispersion measures and multidistances (Martín and Mayor, 2011) or studies on methods of sample quantiles calculation providing nonparametric estimators by implementation in statistical software packages (Hyndman and Fan, 1996). Parameters of dispersion for assessment of postal measurement system efficiency have specific forms reflecting features and purposes of postal operation. These parameters are included and characterized in next parts of this paper.

The main objective of this paper is assessment of measurement system efficiency by calculation of key parameters of dispersion for assessment of measurement results from viewpoint of next modelling. The results of on-time probability estimate accuracy are related to input assumptions, i.e. especially geographical coverage of postal services and geographical stratification on disjunctive set of postal areas. Modelling methods consider one-Operator field of study with set postal transportation network and considering domestic mail flows for various sample sizes.

1 Measuring system characteristic and key parameters of dispersion

The SPPM is collected, processed and delivered by postal operators and measuring process uses representative sample of end-to-end services for addressed mail with set level of service transit time. Design of measuring system includes selection and allocation of test items. These items are posted and received by selected panellists.

Sample design includes specifications of panellists and test items, which must be representative in consideration of design basis. Design basis is the most appropriate structural information available for characterization of real mail distributed in particular field of study.

The result of on-time performance must be expressed as percentage of postal items delivered in transit time just one day after day of posting.

Design of measuring system should ensure representative sample of SPPM test items in the field of study. The most common way to reach representative sample would be design of simple random sample (*SRS*) comprising real mail and then monitoring of its transit time (ČSN EN 13850, 2013). Realization of such design with high accuracy of measuring would be unrealistic. Thus design works with prepared test items sent and received by selected panellists to ensure high quality measuring. This approach to design requires test items to integrate into existing real mail flows with no structural differences. Related structures are created by all characteristics of items with significant influence on result of transit time.

Sample design thus should be stratified according to set of discriminant mail characteristics (*DMC*). Stratification means decomposition of basic statistical set to disjunctive and comprehensive subsets (called strata) considered more homogenous regarding observated characteristics than the whole basic statistical set. Possible characteristics are represented by structures related to senders, receivers, postal logistic network and the whole test items. Postal logistic network can be described by geographical terms and geographical structures should reflect postal network. Characteristics of test items can be based on panellists, induction and delivery points and the whole test items, and must be reflected in measuring structure of panellists, related points and prepared test items. The most frequent discriminant characteristics (with impact on transit time result) are geographical area, type of payment, type of induction, time of posting, formats, weight degrees and method of addressing. Each characteristic has its own possible modes important for results of measuring process and transit time result. Results for strata must be weighted to achieve undeviated result.

Weighted estimation for on-time probability $\hat{p}_{weighted}$ and next related parameters are defined as follows (ČSN EN 13850, 2013):

$$\hat{p}_{weighted} = \sum_{s} \frac{N_s}{N} * \hat{p}_s \qquad (1)$$

where $\hat{p}_s$ is value of on-time probability estimate in stratum *s* and N_s is volume of real mail in geographical stratum *s* (relation between type of postal area of induction/delivery, *N* is volume of real mail). Sample variance of stratified random sample (*StrRS*) is calculated as:

$$\hat{\mathrm{var}}_{StrRS}\left[\hat{p}_{weighted}\right] := \sum_{s} \left(\frac{N_s}{N}\right)^2 \hat{\mathrm{var}}_{SRS}\left[\hat{p}_s\right] = \sum_{s} \left(\frac{N_s}{N}\right)^2 \frac{\hat{p}_s (1 - \hat{p}_s)}{n_s - 1} \qquad (2)$$

where $\hat{\mathrm{var}}_{SRS}\left[\hat{p}_s\right]$ is sample variance of simple random sample *SRS* and n_s (number of test items in stratum *s*) must be higher than *1* for all strata *s*. Then x_s is number of on-time test items in stratum *s* and w_s is weight of stratum *s*.

All strata with any or only one valid test item must be left out from calculation. Weighted results are obtained by mean value $\hat{p}_{weighted}$ considering real mail strata weights *(RSW)* in form of ratio N_s/N, while sample variance is gained by *RSW* square $(N_s/N)^2$.

$$\hat{\mathrm{var}}_{StrRS}\left[\hat{p}_{weighted}\right] = \sum_s w_s^2 \,\hat{\mathrm{var}}_{SRS}\left[\frac{n_s}{n}\hat{p}_s\right] = \frac{1}{n^2}\sum_s w_s^2 \frac{n_s x_s}{n_s - 1}\left(1 - \frac{x_s}{n_s}\right) \tag{3}$$

Sample variance of simple random sample (*SRS*) $\hat{\mathrm{var}}_{SRS}\left[\hat{p}\right]$ is calculated as follows:

$$\hat{\mathrm{var}}_{SRS}\left[\hat{p}\right] = \frac{\hat{p}(1-\hat{p})}{n-1} = \frac{1}{n^2}\frac{n}{n-1} * \left(1 - \frac{x}{n}\right) \tag{4}$$

where $\hat{p}$ is value of on-time probability estimator, n is volume of test items and x is figure of on-time test items.

Sample variance of stratified end-to-end sample (*StrEtE*) is formed as:

$$\hat{\mathrm{var}}_{StrEtE}\left[\hat{p}_{weighted}\right] := \frac{\hat{\mathrm{var}}_{EtE}\left[\hat{p}\right]\hat{\mathrm{var}}_{StrRS}\left[\hat{p}_{weighted}\right]}{\hat{\mathrm{var}}_{SRS}\left[\hat{p}\right]} \tag{5}$$

where $\hat{\mathrm{var}}_{EtE}\left[\hat{p}\right]$ is sample variance of on-time probability estimate in end-to-end measurement system (*EtE*).

Measuring system works with parameter of design factor to reflect covariance and weighing impacts more transparently. In case of *SRS* minimum sample size is calculated directly on the basis of expected or required design accuracy. Sample design should be more sophisticated and complex with consideration of certain principles, when impact of induction and delivery points on measured variation (covariance), stratification by weighting system and appropriate probability distribution for accuracy calculation should be integrated into on-time probability calculation. More complex sample then has in most cases higher level of variance.

Calculation of accuracy for unstratified end-to-end sample shows loss of accuracy due to correlation impacts among test items. These corellations would exist among items sent in the same point of induction, received in the same point of delivery or distributed by relation of the same point of induction-delivery.

The rate for added variance is expressed by design factor *df*, which measures loss of accuracy. It is defined as ratio of sample variance of on-time probability estimator $\hat{p}$ in set sample design to sample variance of on-time probability estimator $\hat{p}$ in *SRS* of the same size. Design factor is always related to set sample design and probability estimator (ČSN EN 13850, 2013).

Accuracy of any end-to-end sample design increases not only by higher volume of items, but also by higher number of senders and/or receivers. Design factor particularly depends on number of used points of induction and delivery. Lower level of design factor is reached by higher number of these points. This fact creates compromise situation between panel size and sample size. Larger panel enables to use more points of induction and delivery and it leads to lower level of design factor and sample size. Design factor thus relates to minimum sample size, which is directly proportional to design factor. It is appropriate to apply small number of test items belonging to concrete point of induction/delivery to reduce corellation impacts.

Design factor parameters are defined by following forms (ČSN EN 13850, 2013). Design factor for end-to-end unstratified measuring system df_{EtE} and for stratified random sample df_{StrRS} are defined as follows:

$$df_{EtE} := \frac{\hat{\mathrm{var}}_{EtE}[\hat{p}]}{\hat{\mathrm{var}}_{SRS}[\hat{p}]} \qquad (6)$$

$$df_{StrRS} := \frac{\hat{\mathrm{var}}_{StrRS}\lfloor \hat{p}_{weighted} \rfloor * (n-1)}{\hat{p}_{weighted} * (1-\hat{p}_{weighted})} \qquad (7)$$

Design factor for end-to-end stratified measuring system df_{StrEtE} is defined in following form:

$$df_{StrEtE} := df_{EtE} * df_{StrRS} = \frac{\hat{\mathrm{var}}_{EtE}[\hat{p}] * \hat{\mathrm{var}}_{StrRS}\lfloor \hat{p}_{weighted} \rfloor * (n-1)}{\hat{\mathrm{var}}_{SRS}[\hat{p}] * \hat{p}_{weighted} * (1-\hat{p}_{weighted})} \qquad (8)$$

2 Modelling of test items sample and results of parameters

Modelling of test items sample is based on parameters of geographical coverage by postal services and stratification of measuring sample for two periods of 2015. Modelling presumes one-Operator field of study with domestic SPPM for various sample sizes in proportion of design basis. Used sample sizes are necessary to cover all postal areas with concrete flows of test items, which must fulfil requirement of proportionality with design basis of real SPPM flows. The first modelling sample considers one-month period with presumed number of 150 panellists, the second one considers two-month period with presumed number of 144 panellists.

Measuring system modelling for both periods is based on relevant indicators figures. These indicators *n, x,* $\hat{p}_{weighted}$ and $\hat{\text{var}}_{EtE}[\hat{p}]$ comprised in Table 1 are necessary data for key dispersion parameters calculation. Applying these input variables, calculated results of dispersion parameters using above-mentioned equations follow:

Tab. 1: Input indicators and dispersion parameters results of modelling

Parameter	1st period	2nd period
n	1086	2096
x	1007	1969
$\hat{p}_{weighted}$	0,928040948	0,939544851
$\hat{\text{var}}_{EtE}[\hat{p}]$	1,16486E-04	5,82863E-05
$\hat{\text{var}}_{SRS}[\hat{p}]$	6,2168E-05	2,716958E-05
$\hat{\text{var}}_{StrRS}[\hat{p}_{weighted}]$	6,0953E-05	2,72279E-05
$\hat{\text{var}}_{StrEtE}[\hat{p}_{weighted}]$	1,14209E-04	5,84115E-05
df_{EtE}	1,873729898	2,145278267
df_{StrRS}	0,990312508	1,004263495
df_{StrEtE}	1,855578155	2,15442465

Source: authors

Calculated results of key parameters of dispersion give satisfactory values. Sample variances have very small values and design factors do not reach high values as well. Results of 1st and 2nd period are naturally influenced by utilization of set measuring system and distribution of test items within panel among others.

Design factor depends on number of test items belonging to point of induction and delivery and also on distribution of test items in existing relations between points of induction and delivery. The number of induction points and distribution are significant points of variation. Stabilization of design factor requires extended design not only with more test items, but also with more induction points.

The number of induction points could be increased to reduce existing correlation of induction points and to minimize design factor. It leads to larger size of senders panel. Due to incorporation of new senders, plan for assignment of test items, which assigns letters to individual relations between points of induction and delivery, could be improved. This improvement could be realized by reduction of existing acummulation of test items in the same relations between point od induction and delivery. It leads to higher distribution of test items in existing relations.

This model designs require approximately *twice* as much test items to reach the same accuracy of *SRS*, which uses each sender and receiver only one time (df_{EtE}). Similar result is given by assumption of *SRS* by calculation of sample size, when concrete design would require in this case also approximately *twice* as much test items to reach the same level of accuracy (df_{StrEtE}).

Using *Z*-multiple of induction points the number of existing relations between points of induction and delivery increases approximately by Z^2-multiple. It follows from this fact, that stabilization factor of panel expansion will lie in range from $(df_{StrEtE} \approx 2)^{1/2}$ – the same volume of letters per relation between points of induction and delivery to $df_{StrEtE} \approx 2$ (the same volume of letters per induction point).

Conclusion

Defined requirements on postal measuring systems within quality of services management, besides main aim of measuring (estimator of transit time of end-to-end services for SPPM), have to meet aims related to accuracy and proportionality to real mail flows including parameters of dispersion. Received figures of parameters by measuring system modelling are sufficiently satisfactory. Values of sample variations are very low and design factors brings relatively good results as well. Mentioned increase of panellists number with corresponding additional number of test items is possible solution, but panel setting and test items distribution to panellist is in practice compromise between expensiveness of measuring system and acceptable results of measurement. Considering applied system of modelled test letter flows, next modelling based on proportional parameters should give reliable results as well.

Acknowledgment

Contractual research „Audit of measurement of the transit time of postal items according to norm ČSN EN 13850 (76 0201)“ for Czech Telecommunication Office

References

Beliakov, G., Pradera, A., & Calvo, T. (2007). *Aggregation Functions: A Guide for Practitioners*. Springer.

Bickel, P. J., & Lehmann, E. L. (1976). Descriptive Statistics for Nonparametric Models (III. Dispersion). *Annals of Statistics*, *4*(6), 1139–1158.

Calvo, T., & Beliakov, G. (2010). Aggregation Functions Based on Penalties. *Fuzzy Sets and Systems*, *161*(10), 1420–1436.

Calvo, T., & Mayor, G. (1999). Remarks on Two Types of Extended Aggregation Functions. *Tatra Mountains Mathematical Publications*, *16*, 235–253.

Gagolewski, M. (2015). Spread Measures and their Relation to Aggregation Functions. *European Journal of Operational Research*, *241*(2), 469–477.

Grabisch, M., Marichal, J.-L., Mesiar, R., & Pap, E. (2011). Aggregation Functions: Means. *Information Sciences*, *181*(1), 1–22.

Huang, Y.-S., Chang, W.-C., & Lin, Z.-L. (2013). Aggregation of Utility-based Individual Preferences for Group Decision-making. *European Journal of Operational Research*, *229*(2), 462–469.

Hyndman, R. J., & Fan, Y. N. (1996). Sample Quantiles in Statistical Packages. *The American Statistician*, *50*(4), 361–365.

Kolacz, A., & Grzegorzewski, P. (2016). Measures of Dispersion for Multidimensional Data. *European Journal of Operational Research*, *251*(3), 930-937.

Martín, J., & Mayor, G. (2011). Multi-argument Distances. *Fuzzy Sets and Systems*, *167*(1), 92–100.

Wilcox, R. R. (2005). *Introduction to Robust Estimation and Hypothesis Testing*. Elsevier.

ČSN EN 13850: Poštovní služby - Kvalita služby - Měření přepravní doby služeb mezi koncovými body pro jednotlivé prioritní zásilky a zásilky první třídy (2013). Úřad pro technickou normalizaci, metrologii a státní zkušebnictví.

Contact

Ing. Daniel Salava, Ph.D.
University of Pardubice – Jan Perner Transport Faculty
Studentská 95, 532 10 Pardubice
daniel.salava@upce.cz

Ing. Kateřina Pojkarová, Ph.D., doc. Ing. Libor Švadlenka, Ph.D.
University of Pardubice – Jan Perner Transport Faculty
Studentská 95, 532 10 Pardubice
Katerina.pojkarova@upce.cz, Libor.svadlenka@upce.cz