

THE USAGE OF MULTICRITERIAL ANALYSIS FOR DETERMINING PRIORITIES AND THE EVALUATION IN TRANSPORT

Jaroslav KLEPRLÍK¹, Bedřich RATHOUSKÝ¹, Markéta BEČIČKOVÁ²

¹Department of Transport Technology and Control,
Jan Perner Transport Faculty, University of Pardubice
²Population Protection Institute

Introduction

To formulate an appropriate optimality criterion for optimizing transport service system is very difficult because there is a conflict of interest among the customer, the transporter operator and the society. Optimization tasks can be solved as monocriterial or multicriterial.

1. Monocriterial optimization

Monocriterial optimizations means solving the problem (task) with only one criterion. This solution is easier but it does not comply with the real situation, on the other hand in some cases it is sufficient. Concerning transport services the customer's criterion K can be e.g. total transportation time. Of course, the passenger wants to minimize it $\Rightarrow \min K$. It is possible to use the graph theory and the algorithm of the shortest path between two points (bus-stops entry and exit) of chosen transportation network, where the points valuation indicate the travel time. The Dijkstra's algorithm described in [1] can be used for finding the shortest path.

2. Multicriterial optimization

Complexity of a real situation brings about certain problems, because using only one criterion is not sufficient. That's why the multicriterial optimization is introduced, which means several criteria to solve the problem. Potentialities for solving the problems are the following:

- to choose the most important criterion and omit the others,
- to use subsequently (in a step-by-step principle) and hierarchically each of the criterion according to their importance \Rightarrow to solve a monocriterial problem primarily, according to the defined importance. If a criterion K_1 is more important than K_2 ; K_2 is more important than K_3 , etc., we first choose a criterion K_1 . If we gain one solution, we think of it as an optimal. If we get a set of multi-element values, we use another criterion of importance from the spectrum of importance K_2 , etc.,
- to create one criterion from partial criteria; so-called „global criterion“. The transfer to a financial valuation is generally used – vide the formula (1).

$$K = f(K_1, K_2, K_3, \dots, K_n), \quad (1)$$

A linear combination (expressed by formula (2)) is also used here.

$$\sum_{i=1}^n c_i * K_i \quad (2)$$

where:

K_ipartial criteria,

c_ipositive coefficients, which express the importance of each partial criteria.

While solving multicriterial optimization of passenger traffic, the conflict among partial criteria takes place. Transport requirements include criteria: hauling distance, speed, travel time, attendance distance, number of crossing passages, ticket price, etc. It is not possible to meet each requirement on the highest level at the same time. The improvement of one criterion may debase the other one. For example the increase of the number of bus stops shortens the walking distance. This is positive aspect, but at the same time it reduces the travelling-speed, and that is unfavorable.

And this raises a question on how to satisfy several, often conflicting requirements at the same time? In such cases we need to choose solution from a number of different variations. It is necessary to find a suitable and acceptable compromise. The multicriterial optimization forms the mathematical model.

We generally have a set of possible variants A which are evaluated by a final number of criteria $f_1 = (a), \dots, f_n(a), a \in A$. We further use criteria, which take positive numeric value and their optimal values will be their maxima. Each criterion therefore forms in A some kind of an arrangement. It is possible that different $a \in A$ responds to the same value of some criterion. There are different approaches for making decisions according to several criteria. The most frequent ones are these:

- Sequential determination of priorities during the process of analyzing a set of alternatives A .
- Aggregation of criteria into one specific function.

Sequential determination of priorities during the process of analyzing a set of the alternatives A

We use the methods, where some parameters of criteria are elicited on the basis of subjective approaches of the problem solver. We assume that f_i are defined on a set $A \subset R^r$. Their border is described by differentiable functions (limitations). The global criterion K , in the quality of function a are supposed to be concave and differentiable. For this it is sufficient for $K(f_1(a), \dots, f_n(a))$ (like the function f_i) to be concave and $f_i(a)$ to be linear on a set A . Or, alternatively $K(f_1(a), \dots, f_n(a))$ (like the function f_i) to be concave and creasing. And all functions f_i to be concave on A .

The algorithm has the following steps:

We assume that $K(f_1(a), \dots, f_n(a))$ is known and we are looking for its maximum on A .

Step 0) by a gradient method we choose starting solution $a_1 \in A$. We put $k = 1$.

Step 1) we establish $b_k \in A$ as a vector face gradient of a composite function $K(f_1(a), \dots, f_n(a))$. Elements of the gradient are partial derivations according to elements a .

Step 2) We find an optimal value of a step t_k as a solution of a maximization $K[f_1(a_k + t_k(b_k - a_k)), \dots, f_n(\dots)]$, we put $a_{k+1} := a_k + t_k(b_k - a_k)$, $k(\text{new one}) := k + 1$ and we optimize Step 1)

We repeat the procedure if in the Step 2) the increase of criterion value is more than predefine value. Or for a priori given number of steps (k value limitation)

In reality we neither know K nor the gradient k of the Step 1). It is probably in the following form (3).

$$\text{grad}_a K[f_1(a), \dots, f_n(a)] = \sum_{i=1}^n \frac{\partial K(a)}{\partial f_i} \text{grad}_a f_i(a) \quad (3)$$

We can use subjective estimates of values $\frac{\partial K(a)}{\partial f_i}$ for the solution, or we can use

the ratios of these values to one of them, e.g. to $\frac{\partial K(a)}{\partial f_1}$. Comparing the results for

various "plausible" variations, we can reach the sub-optimal choice of these weighted coefficients and use them as coefficients in a gradient method, without finding a concrete criterion.

Aggregation of criteria into one specific function

This method is suitable especially when all criteria can be converted to one selected common base. This base could be for example transport-costs, which can be subsequently summed. The final criterion is in the form of (4):

$$K(a) = \varphi_1(f_1(a)) + \dots + \varphi_n(f_n(a)) \quad (4)$$

Frequently, instead of transformation φ_i the multiple of partial criteria values and weighted coefficients v_i can be used, so the global criterion can be expressed in form of (5).

$$K(a) = v_1 \varphi_1(f_1(a)) + \dots + v_n \varphi_n(f_n(a)) \quad (5)$$

If we know that so-called „benefit“ of the variant $a \in A$ according to the criterion f_i can be expressed as (6).

$$\varphi_i(f_i(a)) = c_i f_i(a) + d_i, \quad (6)$$

then c_i and d_i can be determined from knowledge of the best and the worst variation \underline{a}_i a \overline{a}_i , for which the utility u_i from the formula (7).

$$u_i = \varphi_i(f_i(a_i)) \quad (7)$$

assumes its minimum \underline{u}_i and maximum \overline{u}_i , it means from two linear equations with two unknown terms. The final global criterion is the formula (8).

$$K(a) = \sum_{i=1}^n (c_i f_i(a) + d_i) \quad (8)$$

In some cases a conjugated criterion can not to be expressed by the sum (4). That's why the following principle of a conjugated criterion construction expects that for every partial criterion $f_i(a)$ a critical minimum level of m_i exists. It is such a level, that

the values smaller than this m_i are extremely undesirable. Then the optimal a^* will be the one for which the maximum of criterion is achieved according to formula (9).

$$K(a) = \min\{f_1(a) - m_1, \dots, f_n(a) - m_n\} \quad (9)$$

We are looking for $a^* \in A$, such as the non-negative number $k(a^*)$ reaches its maximum, which fulfills the unevenness in the formula (10).

$$f_i(a^*) - m_i - k(a^*) \geq 0, \quad (10)$$

If f_i are linear functions on a linear space of A variants, a linear programming task is concerned.

The task can be expressed in formally similar form, so-called „goal programming“. In this case m_i are entered as required ideal values of functions f_i . Then the suitable global criterion is represented by the weighted summation of absolute values $|f_i(a) - m_i|$ and the weights are designated c_i^+ by $f_i(a) - m_i \geq 0$ and c_i^- by $f_i(a) - m_i < 0$. Left sides of these unevenness's are designated as y_i^+ and y_i^- . Then $f_i - m_i = y_i^+ - y_i^-$, where $y_i^+ \geq 0, y_i^- \geq 0, y_i^+ y_i^- = 0$ and $|f_i - m_i| = \max(y_i^+, y_i^-)$.

If f_i are linear functions, the task of goal programming is equivalent to further linear programming task. The purpose is to find non-negative y_i^+ and y_i^- (for each $i = 1, 2, \dots, n$ where only one can be different from zero), for which it takes the maximum value, expressed in formula (11).

$$\sum_{i=1}^n (c_i^+ y_i^+ + c_i^- y_i^-) \quad (11)$$

under the conditions (12).

$$f_i(a) - m_i + y_i^- - y_i^+ = 0, \quad i = 1, 2, \dots, n \quad (12)$$

The choice of weighted coefficients c_i^+ and c_i^- depends on a concrete setting of a solving task.

The usage of multicriterial analysis for transport services evaluation

One of the used multicriterial analysis methods is a „Method of binate comparing of criteria“. („Juxtaposition criteria method“). For the estimation of weights, it uses information, which of two criteria is more important. A respondent compares every two criteria between themselves in a successive way. The number of juxtapositions can be determined by formula (13).

$$N = \binom{k}{2} = \frac{k * (k - 1)}{2} \quad (13)$$

where:

- N juxtapositions number [figure],
- k criterion, $i = 1, 2, \dots, k$.

The weight of each i criterion can be determined by formula (14).

$$v_i = \frac{n_i}{N} \quad (14)$$

where:

- v_i i criterion weight [-],
- n_i i criterion mark number [figure],
- N juxtapositions number [figure].

Juxtapositions are listed in so-called „Fuller triangle“, whose scheme is in the **Fig. 1**.

1	1	1	1	1
2	3	4	5	k
	2	2	2	2
	3	4	5	k
		3	3	3
		4	5	k
		
			k - 2	k - 2
			k - 1	k
					k - 1
					k

Fig. 1 Fuller triangle scheme

This „Binate comparing of criteria method“ can be used for multicriterial transport services quality evaluation. Quality criteria which influence the demand for public transport include the following points:

- Availability of stops – finding the assessment of suitable attendance distance and finding the attraction area of the stop,

- travel time – the time which is necessary for „door-to-door“ transfer,
- suitable frequency and time allocation of the services – sufficient numbers of services with a sufficient transport capacity at the certain times of the day, week or year (rush period, working day, weekend, summer, winter, holiday season, etc.) directly influence the decision on the usage of the public passenger transport,
- flow of the services within the same transport operator, but also within other operators and different kinds of transport, provision of connection in transport junctions,
- minimum number of crossing passages – customers prefer minimum amount of changes, even if in some cases it means extension of hauling distance or extension of travel time,
- reliability – elimination of irregularities in transportation in connection to timetable adherence (delay minimization, omission of a transport connection, missing the connection),
- regularity of traffic channels – introduction of tact and interval transportation has a very positive reaction among customers (passengers), it eliminates the need to create complicated time tables, which are making difficult time tables which are hard to decipher for most of the passengers,
- adequate transport prices – presently it is one of the most important criterion when deciding whether to use certain kind of transport or a transport system. It is in the interest of the community as a whole to correctly and nondiscriminatory set the prices in both public and individual transport.
- uncomplicated issuance and handling of travel documents – standardization of travel documents, especially offering of integrated travel documents suitable for the whole journey, offering of acquisition discount.
- information availability – not only within the public-transport terminals but also at other places (town information centers, city municipalities, advertisement areas). Informing public well in advance about the timetable changes. Provide information also during the transportation – when to get off, when to cross passage, information about the connections (for example waiting times, post-connections, delay), clear orientation at the terminals and stops - signs orientation (with information about direction to the terminals/stops etc.);
- safety – passengers when choosing the actual mean of transport do not usually take the importance of this criterion into consideration. Level of safety of different means of transport is mostly a subconscious matter. Safety evaluation includes several points - active and passive safety of vehicles, after-accident safety, safety at the stops (getting on/off the means of transport, platform aces), safety concerning criminality etc.,

- travelling culture – comfortable travelling, cleanness of vehicles, stops and stations,
- additional services – may have a positive influence for deciding whether to use particular mean of transport (GSM signal, refreshment, etc.).

The list of evaluative criteria for juxtapositions within multicriterial analysis is for better lucidity summarized in following table **Tab. 1**.

Tab. 1 Evaluative criteria

NO.	CRITERIA
1	Stops availability
2	Travel time
3	Suitable frequency and time distribution of the services
4	Flow of the services
5	Minimum number of crossing passages
6	Reliability
7	Periodicity of traffic channels
8	Reasonable transport price
9	Only one travel-document to use
10	Information availability
11	Safety
12	Travelling culture
13	Additional services

Conclusion

The process of solving optimization tasks can be done by monocriterial or by multicriterial analysis. In the case of multicriterial optimization, the problem is to find the strength of each criterion. There are several ways of solving this problem. First: The method of sequential determination of priorities during the process of analyzing a set of alternatives A. Second: Aggregation of criteria into one specific. Another option is a juxtaposition criteria method (comparing each two criteria between themselves) by respondents. In the case of transport services optimization, after it's evaluation we get a sequel of criteria importance. This is a basis for transport operators and for public

administration for the process of setting priorities which is important for increasing quality of public transport.

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Resumé

VYUŽITÍ MULTIKRITERIÁLNÍ ANALÝZY PRO ZJIŠŤOVÁNÍ PRIORIT A HODNOCENÍ V DOPRAVĚ

Jaroslav KLEPRLÍK, Bedřich RATHOUSKÝ, Markéta BEČIČKOVÁ

Příspěvek se zabývá optimalizačními úlohami v dopravě, které je možno řešit jako monokriteriální či multikriteriální. Především se věnuje multikriteriální optimalizaci, konkrétně postupnému zjišťování priorit v průběhu zkoumání a rozboru množiny alternativ A a dále pak agregaci kritérií do jedné účelové funkce.

Summary

THE USAGE OF MULTICRITERIAL ANALYSIS FOR DETERMINING PRIORITIES AND THE EVALUATION IN TRANSPORT

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This article describes optimization tasks in transport, which can be solved by monocriterial or multicriterial way. The article addresses mainly a multicriterial optimization, especially with Sequential determination of priorities during the process of analyzing a set of alternatives A and it also deals with aggregation of criteria into one specific function.

Zusammenfassung

VERWENDUNG VON DER MULTIKRITERIALEN ANALYSE FÜR ERMITTLUNG DER PRIORITÄTEN UND BEWERTUNGEN IM VERKEHR

Jaroslav KLEPRLÍK, Bedřich RATHOUSKÝ, Markéta BEČIČKOVÁ

Der Beitrag befasst sich mit Optimierungsaufgaben im Verkehr, die als monokriterial oder multikriterial gelöst werden können. Hauptthema des Beitrags betrifft die multikriterialen Optimierung. Konkrete Beispiele sind vor allem sukzessive Ermittlung der Prioritäten im Laufe der Untersuchung und Analyse der Alternativmenge und Gruppierung der Kriterien in eine Zielfunktion.