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BASIC CONTACT LINE ELECTRICAL PARAMETERS OF THE ELECTRIFIED TRACK

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1. Introduction

The feeding system of electrified railways consists of two parts, i. e.:

- contact line (catenary), trolley and catenary messenger,
- rails and ground (soil, earth).

The magnetic field of the feeding systém is caused with the closed loop inductance L, the electric field is caused with the open loop capacity C.

The specific inductance L/km is dependent on frequency. The ***Fig. 1*** presents this relation in the case of 13 km single line length measured with power amplifier.

It is possible to calculate the specific capacity C/km, because it is independent on frequency. The usefull expression of trolley is

$$\frac{C_T}{\ell} = 2\pi\varepsilon_0 \cdot \frac{ln\frac{2h_2}{r_2} - ln\frac{b}{a}}{ln\frac{2h_1}{r_1} \cdot ln\frac{2h_2}{r_2} - ln^2\frac{b}{a}} \quad [F/km] \qquad (1)$$

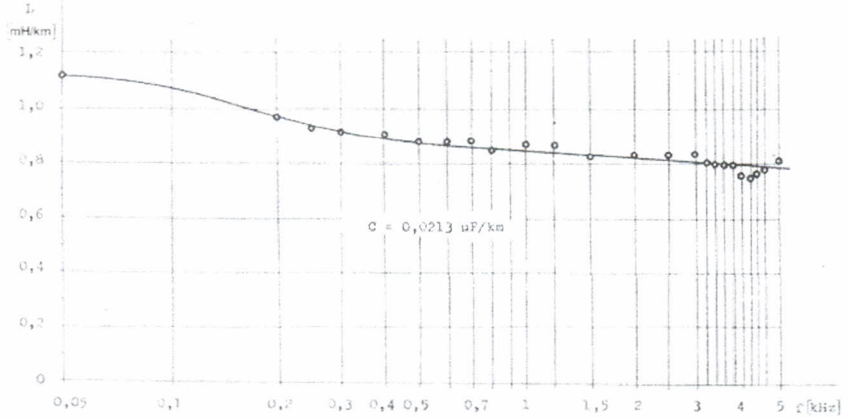

Fig. 1

The specific capacity of messenger is

$$\frac{C_M}{\ell} = 2\pi\varepsilon_0 \cdot \frac{\ln\frac{2h_1}{r_1} - \ln\frac{b}{a}}{\ln\frac{2h_1}{r_1}\cdot\ln\frac{2h_2}{r_2} - \ln^2\frac{b}{a}} \quad [\mathrm{F/km}] \qquad (2)$$

In this exprossion:

$h_1 \cong 5{,}6$ m $\cong$ b........................trolley hight above ground
$h_2 \cong 6{,}6$ m..............................messenger hght above ground
$r_1 \cong 0{,}005$ m ....trolley radius
$r_2 \cong 0{,}005$ m ....messenger radius
$a \cong 1$ m ....contact line hight
$\varepsilon_0 = 8{,}855 \,.\, 10^{-12}$ F/m............vacuum permitivity

The total calculated capacity of the trolley, messenger and 167 hangers (1,191 nF) with these parameters is $C \cong 5{,}925 + 5{,}766 + 1{,}191 = 12{,}882\ nF/km$.

Remark: the contact line in ***Fig.1*** is eked out with a booster line.

2. Modes

Three modes are applicable for calculations in the field of railway feeding system, especially for the analyses of voltages and currents in the catenary system:

A: it is posible to consider the contact system like lossy long line with distributed specific induktivity L/km, capacity C/km and resistances (transverse G/km and longitudinal R/km),

B: the next step is to use of two-port network series,

C: the simpliest method is to replace the actual contact line by concentrated inductivity L (if the line is shortcircuited) or by concentrated capacity (if the line is open).

Each of these three modes have specific advantages and inconveniences, which are analysed in this paper hereafter.

A. The use of lossy line theory

This approach is well known and suitable for calculations. In actual conditions there are some simplification acceptable based on the real conditions of the railway contact line (catenary):

- the length of the contact line between railway stations is substantially greater in comparison with the length of the contact line at railway station,
- the geometry of the contact line is lengthwise mostly changeless,
- the points where the wave impedance alternates are sporadic (at railway stations, at substations),
- the soft-ware for lossy line analyses is available to any user with good results.

Well known expressions for wave impedance and wave propagation speed are:

$$Z_W = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{3}$$

$$\gamma = \sqrt{(R + j\omega L) \cdot (G + j\omega C)} = \beta + j\alpha \ . \tag{4}$$

Next usefull expressions are for:

- imput impedance of open lossy line

$$Z_0(f) = Z_W \cdot cotgh\,(\gamma \cdot \ell) \tag{5}$$

- imput impedance of short circuited lossy line

$$Z_K(f) = Z_W \cdot tgh\,(\gamma \cdot \ell) \ . \tag{6}$$

Further simplification is acceptable in some calculations, based on specific R and G neglection, aspecially for traction system 25 kV, 50 Hz. The parameter R could be neglected because the currents in the contact line have usually very small value. The neglection of the parameter G is today acceptable due to modern insulators based on composite materials.

Under these siplifications ($\beta \cong 0$) two expressions are common:

- imput impedance of open contact line

$$Z_0(f) \cong -jZ_W \cdot cot\,g\,(\alpha \cdot \ell) \qquad (7)$$

- imput impedance of short circuited contact line

$$Z_K(f) \cong jZ_W \cdot tg(\alpha \cdot \ell)\ . \qquad (8)$$

The symbols in these expression describe:

$$Z_W \cong \sqrt{\frac{L}{C}}\quad [\Omega]\ \text{the wave impedance} \qquad (9)$$

$$\alpha \cong 2 \cdot \pi \cdot f \cdot \sqrt{L \cdot C}\quad [rad\,/\,km]\ \text{the wave propagation speed.} \qquad (10)$$

The most common base values of railway contact line are:

L ≅ 1,2 – 0,8 mH/km ....dependent on frequency (***Fig. 1***),

C ≅ 15 – 20 nF/km ....undependent on frequency.

These expressions are applicable for example at calculations of resonance frequencies, where the damping effect could be neglected.

The mode of using lossy line theory is contrariwise not acceptable for building of an laboratory physical model of railway contact line. The ground lies in a very hight actual contact line parameters quality, which is unpossible to keep with discrete coils and capacitors at acoustic (harmonic) frequencies. In the field of ultra high frequency the Lecher-line could create an usefull laboratory model for demonstration of uhf waves.

Fig. 2 presents the measured input impedances of contact line (13 km) in open („vedení naprázdno“) and short circited („zkratované vedení“) conditions, for frequnces up to 5 kHz.

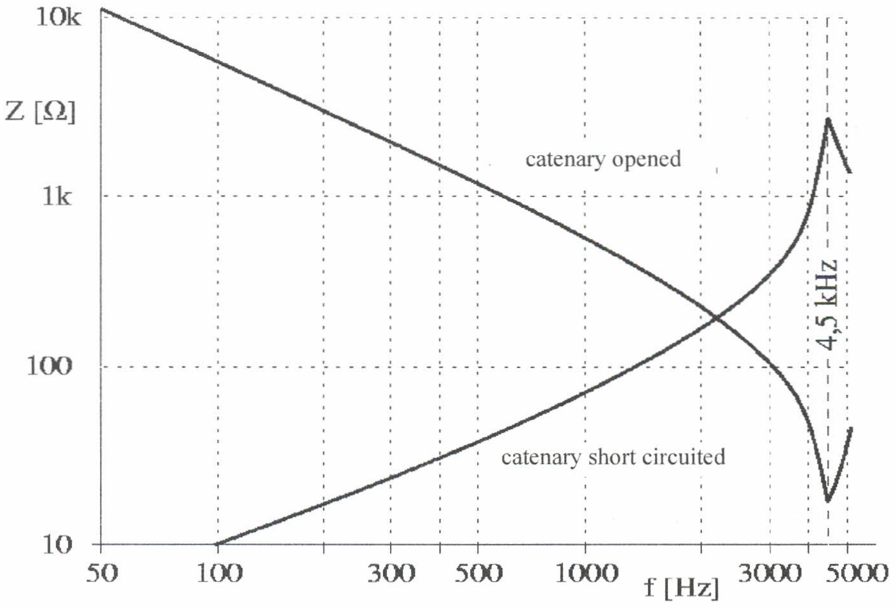

Fig. 2

Very important is the point with frequency approx. 4,5 kHz. This frequncy value is the eigen frequncy of the contact line given from the expression

$$\alpha \cdot \ell = \frac{\pi}{2} \quad \triangleright \quad f = \frac{1}{4 \cdot \ell \cdot \sqrt{L \cdot C}} \quad . \tag{11}$$

B. The use of two-port network

The theory of two-port networks is in detail an sohpisticated system, derived for historical purpose, especially for telecommunication. Due to PC computation technique the denotation of this mode depreciate.

On the other hand this mode describes the possibility to construct laboratory model of railway contact line. The most valuable is the appreciation of frequency restriction at prescribed fault rate.

According to actual parameters of railway contact line how they are there is possible to accept this conditionally

$$|\gamma| \cdot \ell << 1 \quad . \tag{12}$$

Under this condition it is possible to replace the contact line by an single two-port network. From the mathematical viewpoint this expression could be used for simplification of (4) with an result

$$tgh(\gamma \cdot \ell) \cong \gamma \cdot \ell \quad . \tag{13}$$

Receiving the terminal threshold of calculation fault rate „-1,5 %“ we are able to write

$$\ell_{MAX} \cong \frac{0{,}03 \cdot \omega_{MAX}}{\alpha_{MAX} \cdot f_{MAX}} = \frac{0{,}03}{f_{MAX} \cdot \sqrt{L \cdot C}} \tag{14}$$

or

$$f_{MAX} = \frac{0{,}03}{\ell_{MAX} \cdot \sqrt{L \cdot C}} \quad . \tag{15}$$

The expression (11) gives the greatest value for the contact line length, which is possible to replace with one Π or T - two-port network under the condition of fault we have mentioned above.

For example: L = 1 mH/km, C = 15 nF/km, ℓ = 13 km. Under these conditions the greatest frequency, which could be analysed by one Π or T – two-port network, at the given railway contact line section, is only 596 Hz.

If we need to analyse up to 2 kHz, we have to install at least 4 two-port networks. Each two-port network could replace 3,25 km of the contact line. The components of each Π or T – two-port network have values: L = 3,25 mH, C = 0,04875 μF.

Such a model of railway contact line with Π – two-port network applicable for 16 2/3 Hz and model voltage 7 V is described in [2]. The feeder line length is 38 km. Electrical parameters of the railway contact line are: L = 1,24 mH/km, C = 32 nF/km, R = 0,088 Ω/km. For achievement the prescriebed fault rate there were 6 Π – two-port networks necessary. It meens, that each Π – two-port network represents approximately 6 km. In accordance with the expression (15) the greatest frequency for analyses is approx. 800 Hz.

The L- value of the described Π two-port network has necessity of modelling the effect of frequency dependent earth resistance, too. The solution gives the ***Fig. 3***.

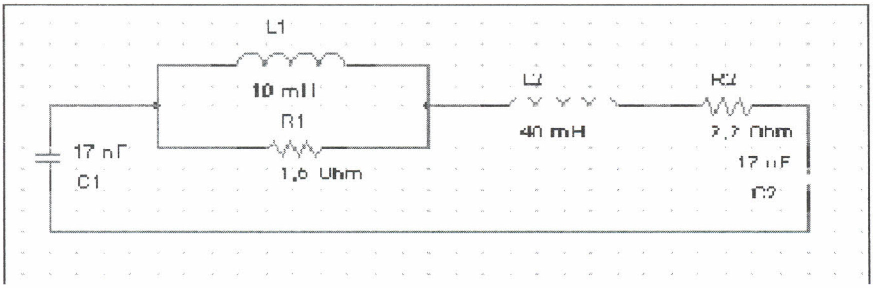

Fig. 3

C. The use concentrated inductivity L (if the line is shortcircuited) or concentrated capacity (if the line is open)

This mode of modelling the contact line is applicable only for simpliest calculations, for example in the case of design of power factor correction equipment, i. e. up to 7. harmonic component.

If the line is one-sided shortcircuited, the input impedance is given in the modified expression (8)

$$Z_K(f) \cong jZ_W \cdot tg\left(2 \cdot \pi \cdot f \cdot \ell \cdot \sqrt{L \cdot C}\right) \;. \tag{16}$$

The input impedance of the substitute induktivity L_{SUBST} is

$$Z_{K,SUBST} = 2 \cdot \pi \cdot f \cdot \ell \cdot L_{SUBST} \;. \tag{17}$$

The function *tg* has a progression

$$tg(\chi) \cong \chi + \frac{\chi^3}{3} + \frac{2\chi^5}{15} + etc. \tag{18}$$

$$\text{where } \chi = 2 \cdot \pi \cdot f \cdot \ell \cdot \sqrt{L \cdot C} \;. \tag{19}$$

After some formating we have the expression

$$Z_K(f) \cong Z_{K,SUBST} + \frac{(2 \cdot \pi \cdot f \cdot \ell)^3 \cdot L^2 \cdot C}{3} + etc. \;. \tag{20}$$

The substitution error due to concentrated induktivity is approx. 6,11 %.

In like manner we are able to replace the open contact line by concentrated capacity C_{SUBST} .

If the line is open, the input impedance is given in the modified expression (7)

$$Z_0(f) \cong -jZ_W \cdot cot\,g\left(2 \cdot \pi \cdot f \cdot \ell \cdot \sqrt{L \cdot C}\right) \;. \tag{21}$$

The function cotg has a progression

$$\cot g(x) \cong \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - etc. \quad . \qquad (22)$$

The input impedance of the substitute capacity C_{SUBST} is

$$Z_{0,SUBST} = \frac{1}{2 \cdot \pi \cdot f \cdot \ell \cdot C_{SUBST}} \quad . \qquad (23)$$

After some formating we have the expression

$$Z_0(f) \cong Z_{0,SUBST} - \frac{2 \cdot \pi \cdot f \cdot \ell \cdot L}{3} - etc. \quad . \qquad (24)$$

The substitution error due to concentrated capacity is approx. - 6,52 %.

These both substitution errors are acceptable in comparision with accuracy of input railway contact line measured parameters.

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Resumé

ZÁKLADNÍ ELEKTRICKÉ PARAMETRY TRAKČNÍHO VEDENÍ ELEKTRIZOVANÉ TRATI

Karel HLAVA, Ladislav MLYNAŘÍK

Článek shrnuje základní elektrické parametry trakčního vedení z hlediska možného zjednodušení pro některé výpočty. Jsou analyzovány tři typy náhrady trakčního vedení, a to homogenní vedení, dvoubrany a náhrada jednoduchým L či C. Výpočty založené na teorii homogenního vedení se jeví jako nejvhodnější, použití teorie dvoubranů je vhodná pro návrh laboratorního modelu trakčního vedení. Třetí způsob je vhodný pro výpočty až do zhruba 400 Hz.

Summary

BASIC RAILWAY CONTACT LINE ELECTRICAL PARAMETERS OF THE ELECTRIFIED TRACK

Karel HLAVA, Ladislav MLYNAŘÍK

The paper summarizes basic electrical parameters of the contact line from the viewpoint of possible simplification at particular calculation. Three modes are discussed, e.g. homogeneous line, two-port network and substitution with simple L or C. The calculations based on homogeneous line theory appear as the most preferable, the use of two-port network theory appears preferable for contact line design of labor model. The third mode is adequate for calculations up to approx. 400 Hz.

Zusammenfassung

ELEKTRISCHE GRUNDPARAMETER DER FAHRLEITUNG ELEKTRIFIZIERTER STRECKEN

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Der Artikel fasst die Grundparameter der Fahrleitung zusammen aus dem Gesichtspunkt der möglichen Vereinfachung für einzelne Berechnungen. Man diskutiert drei Mőglichkeiten, und zwar eine homogene Leitung, Vierpole und Substitution mit einfache L oder C. Es scheint, daß die Berechnungen gegründene auf der homogenen Leitung sind günstigste, die Vierpoltheorie eignet sich am besten für Konstruktion eines Labormodells der Fahrleitung. Die dritte Mőglichkeit kann man in Berechnungen bis cca 400 Hz benützen.