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SPHERICAL PARTITION OF ROAD TANK SUBJECTED TO EXTERNAL PRESSURE

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1. Introduction

The aim of this article is to perform a detail computational analysis of a spherical partition welded into a road beer-tank. The joint of the cylindrical shell and the partition is stiffened by circumferential ring. Both cases of external ring and internal ring are considered. The partition is subjected to an external pressure. The fully non-linear stability analysis in which material and geometrical nonlinearities are taken into consideration is supposed. The material nonlinearity enables a study of the limit state of plasticity while the geometrical nonlinearity displays the possible loss of stability. The article is complemented by a brief reflection on the impact of initial imperfections on stability of strongly non-linear thin-walled shell structures.

2. Theoretical background

The loss of stability is one of the limit states which may occur in excessively loaded thin-walled structures. It is proved in the shell theory that they can collapse in various ways depending on geometrical parameters, boundary conditions, loading conditions, material characteristics and initial imperfections. A stability collapse is induced by minimum load corresponding to a particular form of deformation. The membrane energy is converted to both the membrane and bending energy. As the membrane stiffness of the shell structures is several orders higher than the bending stiffness, the loss of stability is attended by large displacements of a wave character often visible to the naked eye.

Bushnell D. describes in his book Computerized Buckling Analysis of Shells [1] basic types of loss of stability:

- linear buckling,
- nonlinear axially symmetrical collapse,
- nonlinear buckling.

Linear buckling (point B_L in ***Fig. 1***) is characterized by a deformation of the structure in a new shape entirely different from the pre-buckling shape. The structure collapses when the curve tangent behind the bifurcation point is negative. The critical load corresponding to the bifurcation point is computed by means of the generalized eigenvalue theory.

Nonlinear axially symmetrical collapse (point A in ***Fig. 1***) is characterized by decreasing stiffness of the structure with increasing load. In most cases, when the peak of the equilibrium curve is reached, the sudden loss of stability follows. The cap snaps through into its inverse position. The snap-through occurs in an axially symmetrical form along the curve 0AC.

Nonlinear buckling is characterized either by axially nonsymmetrical snap-through along the curve $0B_{N1}D$ or by nonlinear axially symmetrical collapse subsequently followed by axially nonsymmetrical snap-through along the curve $0AB_{N2}E$.

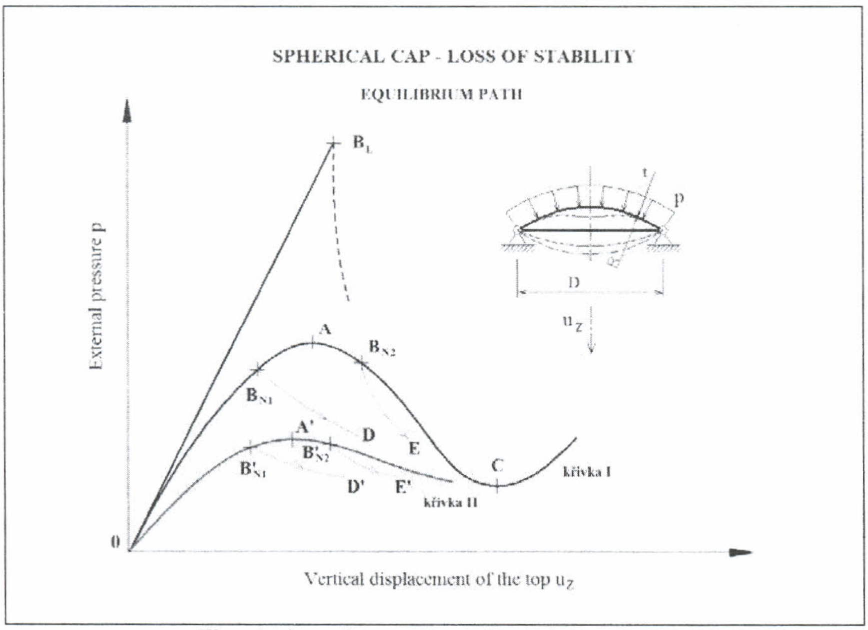

Fig. 1 *Loading path of the spherical cap*

The real structures with initial imperfections exhibit similar behavior, however, the loading curve (curve II) is lower than the loading curve valid for the *ideal structures* (curve I). The collapse occurs again due to either the nonlinear buckling or nonlinear axially symmetrical collapse. Nonlinear axially symmetrical collapse followed by nonlinear buckling is also possible.

3. Spherical partition of road tank

The following text is devoted to a numerical analysis of a spherical partition inside a road beer-tank (see ***Fig. 2*** to ***Fig. 4***). The partitions divide the tank into three separate sections. The partitions are currently loaded either from inner or outer side (strength or stability problem). Based on the producer's information, the spherical partition sometimes loses stability and snaps-though into its inverse position. The cylindrical shell close to the collapsed partition visibly deforms in elastic-plastic area. This phenomenon is not permitted by standards for pressure vessels.

Basic parameters:

R = 1200 mm ....radius of curvature of partition,
D = 1400 mm........................internal diameter of cylindrical shell,
t_{1n} = 8 mm ....nominal thickness of partition,
t_{2n} = 4 mm ....nominal thickness of cylindrical shell,
c_1 = 0,9 mm..........................total thickness allowance of partition,
c_2 = 0,3 mm..........................total thickness allowance of cylindrical shell,
A = 1200 mm^2.......................cross-sectional area of outer stiffening ring (50x24 mm).

The thickness is reduced by allowances:

$$t_1 = t_{2n} - c_1 = 8 - 0{,}9 = 7{,}1\ \text{mm} \tag{1}$$

$$t_2 = t_{2n} - c_2 = 4 - 0{,}3 = 3{,}7\ \text{mm} \tag{2}$$

Length of the cylindrical shell on both side of the partition is:

$$L_1 = L_2 = 2{,}5 \cdot \sqrt{R \cdot t} = 2{,}5 \cdot \sqrt{1200 \cdot 4} = 173{,}2 \approx 200\ \text{mm} \tag{3}$$

Material:

The structure is designed of austenitic stainless steel 14 301 with material properties at ambient temperature 20°C [6]. Von Mises's elastic-plastic bilinear model of the used material is considered:

$E = 2.10^5$ Nmm^{-2}..................Young's modulus of elasticity,
μ = 0,3 ....Poisson's number,
f_y = 250 Nmm^{-2} ....yield strength,
$E_T = E / 10^4$ ....tangent modulus.

Fig. 2 Track with road beer-tank

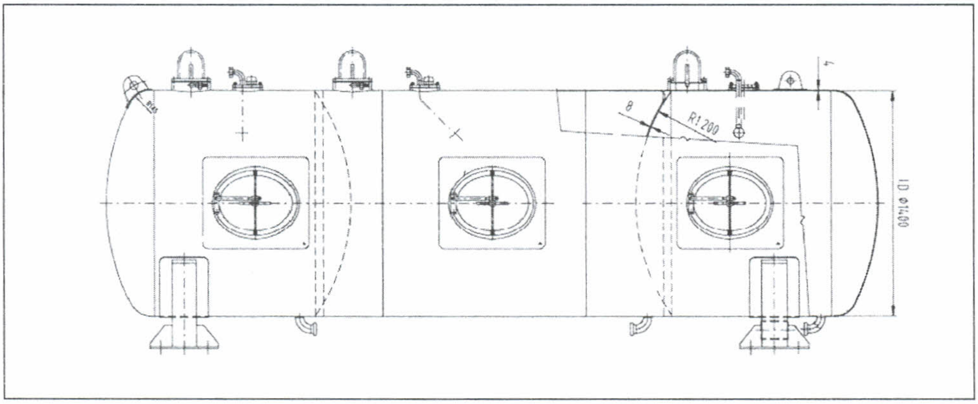

Fig. 3 Scheme of road beer-tank with spherical partitions

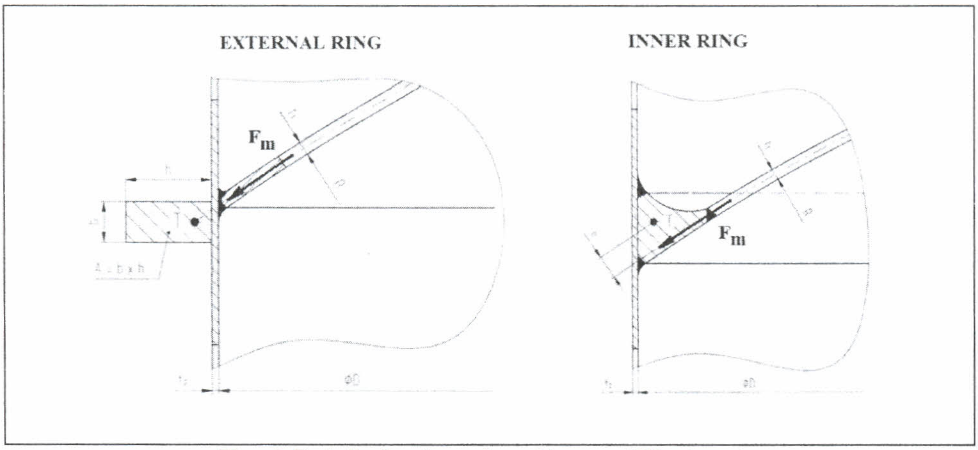

Fig. 4 Detail of external and inner stiffening ring

Load:

The maximum outer/inner design pressure is $p = 0,3$ MPa. The hydrostatic pressure is neglected.

3.1 Analysis – SHELL4T elements

The finite element model (FEM model) of the structure with the outer stiffening ring (see ***Fig. 4*** and ***Fig. 5***) is created for the purpose of computer analysis. The model is assembled of 3076 elements SHELL4T by means of computer program COSMOS/M [7]. The cylindrical coordinate system *Oxyz* with radial direction *x*, circumferential direction *y* and axis *z* identical with the axis of the cylindrical shell is defined. The ideal structure without initial imperfections is considered. The nonlinear computational analysis is governed by Riks's arc-length control procedure.

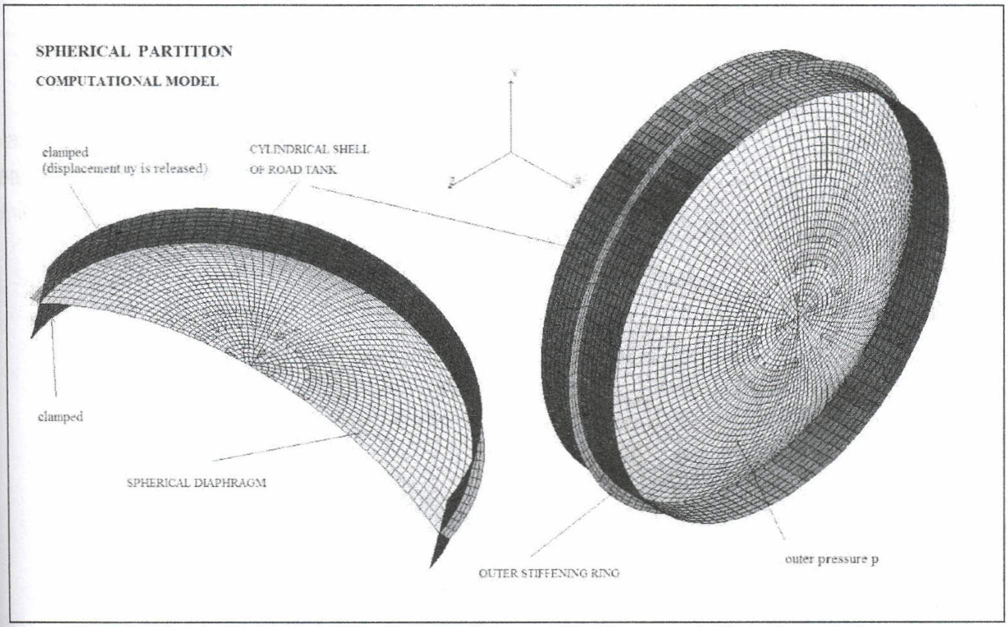

Fig. 5 *Partition with external ring – shell computational model*

The equilibrium path is shown in ***Fig. 6***. It represents the ratio between loading pressure p and displacement u_z in the middle of the model. Initially the model performs a linear behavior. At pressure $p \approx 1,3$ MPa the stiffness begins to fall due to a non-linear nature of the structure. The plasticity may contribute to this phenomenon. The non-linear axially-symmetrical collapse occurs in point A (step 12 of the non-linear computational procedure) with limit pressure $p_L = 1,383$ MPa. The non-linear collapse of the model into its inverse position is starting. However, the axially symmetrical collapse is disturbed in point

B by asymmetric non-linear buckling. The deformed shapes in points A and B are shown in ***Fig. 7***. The analysis is completed in step 1000 where the spherical partition is in its inverse position (see ***Fig. 8***). The radial displacements u_x indicate a local loss of stability of the compressed stiffening ring.

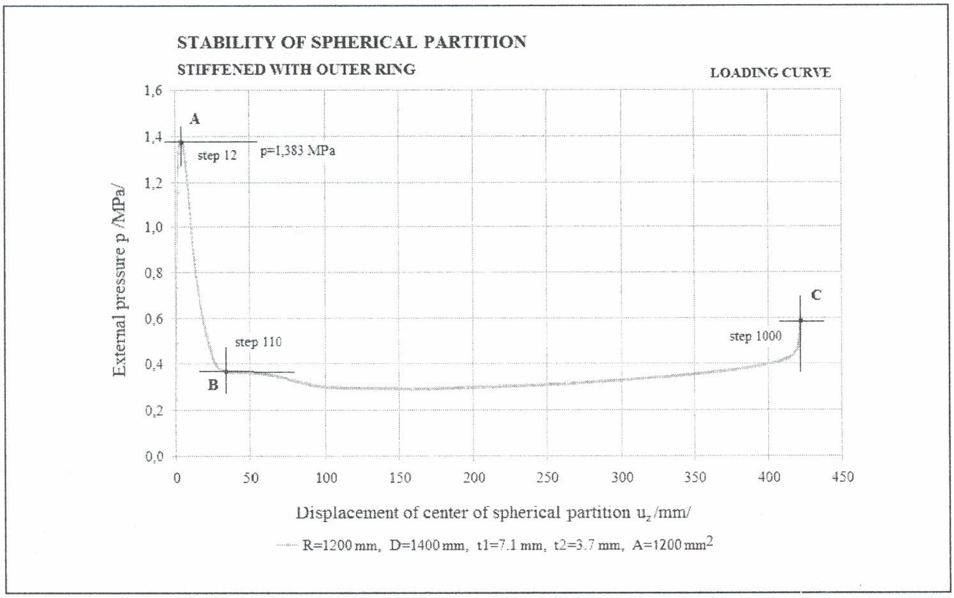

Fig. 6 *Equilibrium path (loading curve)*

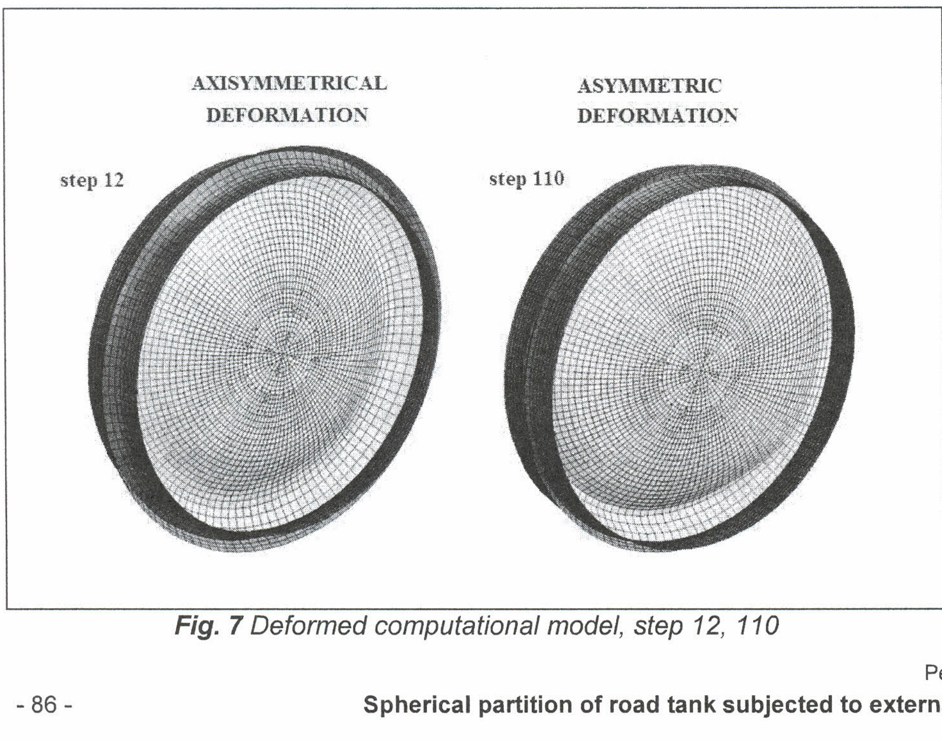

Fig. 7 *Deformed computational model, step 12, 110*

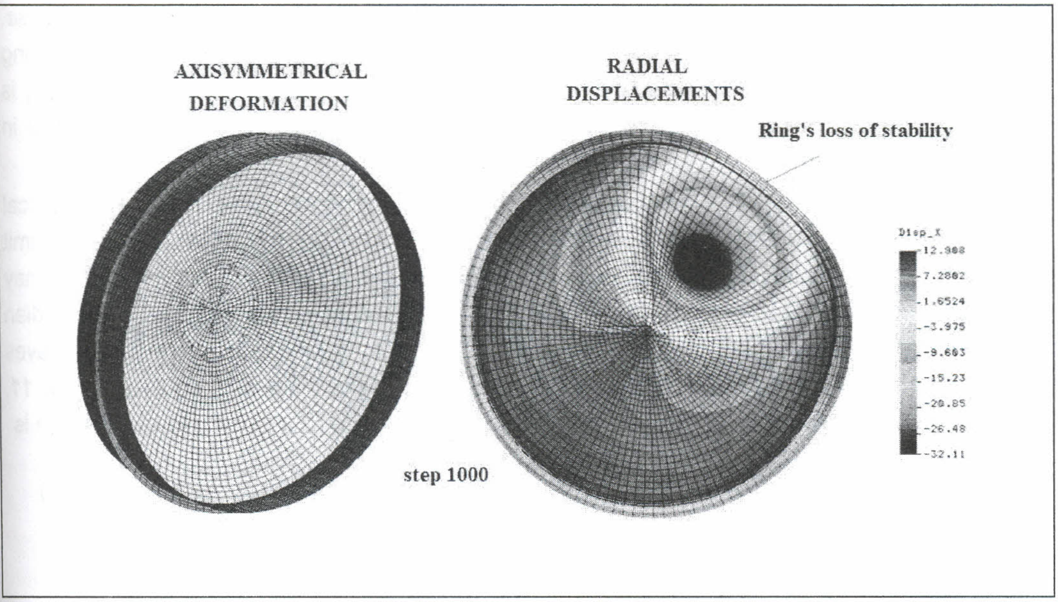

Fig. 8 *Deformed computational model, step 1000*

The limit pressure p_{PL}=1,383 MPa (step 12) is a result of the computational elastic-plastic stability analysis of the ideal structure without initial imperfections. Stability of the real structure is determined based on the European Recommendation ECCS [2]. The limit pressure of the real structure is:

$$p_u = \frac{\alpha_0 \cdot p_{LP}}{\gamma} = \frac{0{,}506 \cdot 1{,}383}{1{,}333} = 0{,}525 \text{ MPa} \tag{4}$$

where safety factor γ = *4/3* and reduction factor α_0 is for ratio

$$\frac{R}{t_1} = \frac{1200}{7{,}1} = 169 < 212 \tag{5}$$

$$\alpha_0 = \frac{0{,}83}{\sqrt{1 + 0{,}01 \cdot R / t_1}} = \frac{0{,}83}{\sqrt{1 + 0{,}01 \cdot 1200 / 7{,}1}} = 0{,}506 \tag{6}$$

Stability condition is prescribed in a form

$$p = 0{,}3 \text{ MPa} \le p_u = 0{,}525 \text{ MPa} \qquad \text{Fulfilled !}$$

On the basis of the results may be said that spherical partition of the road tank is properly designed and withstands the prescribed load.

3.2 Analysis – PLANE2D body of revolution elements

Now, an inner ring of special shape (see ***Fig. 4***) is additionally designed due to a better sanitation of the tank. The ring has approximately the same cross-sectional area like the outer ring considered in the previous analysis. The previous analysis shows that the shell computational model snaps-through in a form of axially symmetrical collapse. Based on this fact, a simplified computational stability analysis may be performed using PLANE2D-body of revolution elements. The computational model with the inner ring is shown in ***Fig. 9***. The lower end of the model is clamped while the upper end is free in *y* direction. The model is loaded by external pressure *p*.

The equilibrium curve represents a relationship between pressure *p* and vertical displacements u_y of the model's center (see ***Fig. 10***). The model collapses at limit pressure p_L = 0,896 MPa. This relatively low value compared with the previous case may be explained by the bending moment which is a product of eccentricity *e* and meridian forces F_m (see ***Fig. 4***). The bending moment disturbs the membrane state and behaves as an initial imperfection. The reduced stresses in the limit state are shown in ***Fig. 11***. The extensive plastic area of the ring is evident. The limit pressure of the real structure is:

$$p_u = \frac{\alpha_0 \cdot p_{LP}}{\gamma} = \frac{0{,}506 \cdot 0{,}896}{1{,}333} = 0{,}340 \text{ MPa} \tag{7}$$

Stability condition is prescribed in a form:

$$p = 0{,}3 \text{ MPa} \leq p_u = 0{,}340 \text{ MPa} \qquad \text{Fulfilled !}$$

On the basis of the results might be said that spherical partition with the special inner ring is properly designed and withstands the prescribed load.

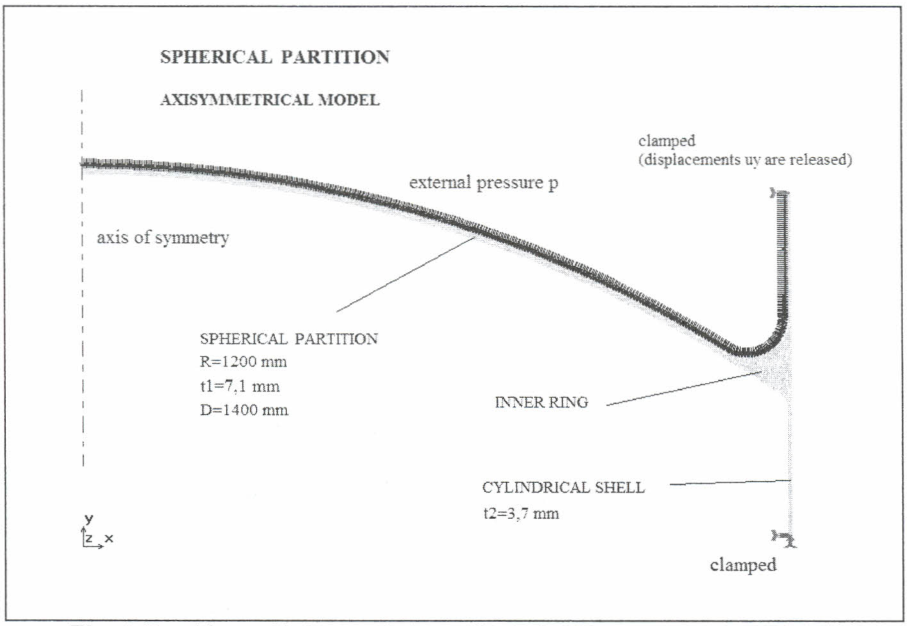

Fig. 9 *Partition with internal ring – PLANE2D computational model*

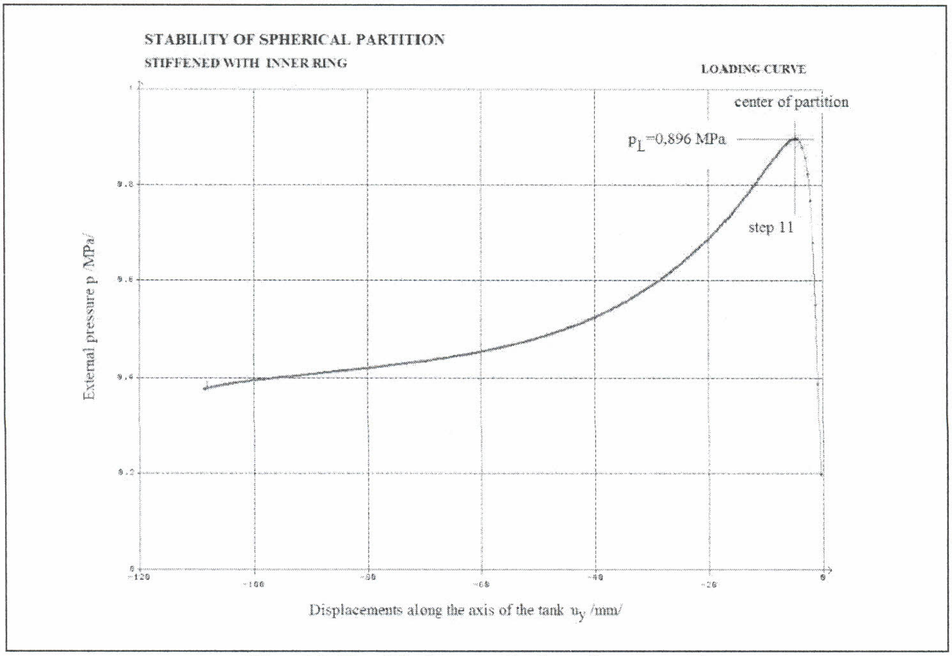

Fig. 10 *Equilibrium path (loading curve)*

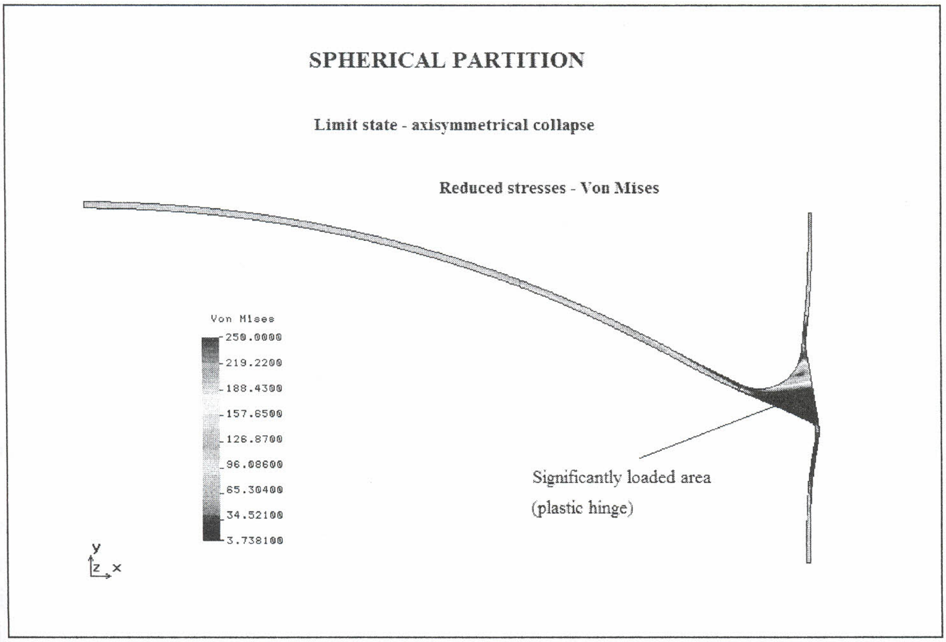

Fig. 11 *Axially symmetrical collapse – stresses in limit state (step 11)*

4. Conclusion

The first analysis of the shell computational model (see ***Fig. 5***) proved that the structure loaded by external pressure starts to collapse in an axially symmetrical shape. Based on this fact, a simpler analysis of the model created of body of revolution elements is recommended (see ***Fig. 9***). Comparing the limit pressures of both analyzed models, the first model shows 54% higher carrying capacity than the second one. It can be explained by the additional bending moment $M = F_m \cdot e$ which disturbs the membrane state of stress and behaves as an initial imperfection. This moment is a product of the eccentricity e and the meridian forces F_m (see ***Fig. 4***). Regardless the difference between the results, it might be said that the spherical partition of the road beer-tank is properly designed and withstands the prescribed load.

The thin-walled spherical caps have significant use in practice namely in the process and energy industry. They are used, for example, for a production of pressure vessels, storage tanks, bins, etc. Designers often encounter problems with *stability* of excessively loaded thin-walled shell structures. The spherical cap with the flexible edge such as spherical partition inside a road tank is a *strongly non-linear problem*. Commonly used standards and recommendations for stability calculations [2], [3], [4], [5], [6] do not provide convenient analytical procedures for these nonlinear cases. Therefore, a numerical computer analyses often supported by experiments are necessary. Stability of thin-walled shells is significantly influenced by the *initial imperfections* which induce, in addition to membrane stresses, also bending stresses in pre-buckling stage of loading. The standards and recommendations only provide procedures for stability calculations of standard structural parts with a relatively linear pre-buckling behavior (cylindrical shells, spherical shells, etc). The initial imperfections of real structures are included in the calculation by means of the *reduction factor* α_0 derived from a large number of experiments. The limit load of the *ideal structure* is then transferred to the limit load of the *real imperfect structure* by means of α_0.

In case of the spherical partition with flexible edge in radial direction, the sufficiently accurate results cannot be expected. The initial imperfection sensitivity decreases with a decrease of the radial stiffness at the edge. Therefore, using the standard reduction factor α_0, results of more flexible spherical partition can be considerably conservative.

The arguments described above clearly shows a need to study the problem of initial imperfections of strongly non-linear thin-walled shell structures in detail. The supposed ratio α_0 versus the parameter R/t for various boundary conditions is shown in ***Fig. 12***. The lower thick curve is prescribed by ECCS [3] for axially compressed cylindrical shells or for spherical shells loaded by external pressure. The remaining curves are the preliminary estimation. The problem of initial imperfections will be a subject of further research.

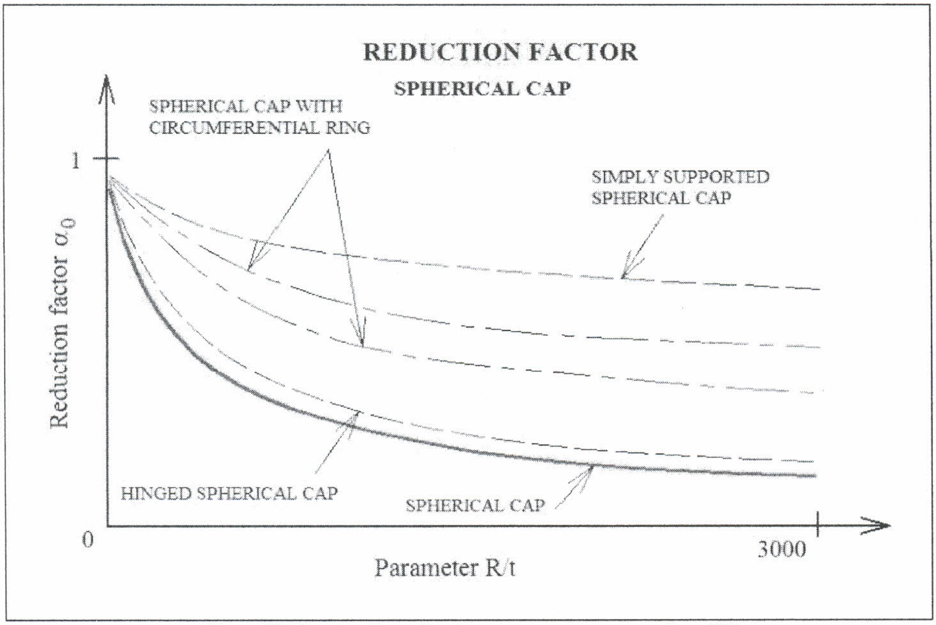

***Fig. 12** Reduction factor versus parameter R/t*

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References

1. BUSHNELL D. *Computerized buckling analysis of shells.* Martinus Nijhoff Publishers. Dordrecht. ISBN 90-247-3099-6 (1985).
2. DIN 18800, part.4. *Structural steelwork. Analysis of safety against buckling of shells. German norm*. Beuth Verlag GmbH, Berlin (1990).
3. ECCS-*Buckling of Steel Shells.* European Design Recommendations. Fourth edition, Published by ECCS (1988).
4. ECCS-*Buckling of Steel Shells*. European Design Recommendations. Fifth edition, Published by ECCS (2008), *ISBN* 92-9147-000-92.
5. EN 1993-1-6. *Design of steel structures – Part 1-6: Strength and Stability of Shell Structures*. European Committee for Standardization (2007).
6. EN 10028-7. *Flat products made of steel for pressure purposes – Part 7: Stainless steels*, 2000.
7. FEM Computer program *COSMOS/M, Version 2.95* by SRAC (Structural Research and Analysis Corporation), Los Angeles, California.

Resumé

VRCHLÍKOVÁ PŘEPÁŽKA SILNIČNÍ CISTERNY ZATÍŽENÁ VNĚJŠÍM TLAKEM

Petr PAŠČENKO

Tenkostěnný kulový vrchlík zatížený vnějším tlakem vykazuje nelineární chování. Tento jev je ještě zesílen v případě prostě podepřeného vrchlíku s volným posuvem okraje v radiálním směru. Zde již není možné pro výpočet mezního tlaku použít běžných výpočtových postupů na bázi lineární mechaniky. Je třeba provést detailní numerickou počítačovou analýzu využívající např. metodu konečných prvků (MKP). Kromě velkých posuvů lze zohlednit i nelineární chování materiálu. Déle je třeba si uvědomit, že stabilitní únosnost může být značně ovlivněna počátečními imperfekcemi. Použití standardních postupů při výpočtu reálných kulových vrchlíků je sice možné, avšak často značně konzervativní. Otázka vlivu počátečních imperfekcí na stabilitu kulových vrchlíků je předmětem dalšího výzkumu.

Summary

SPHERICAL PARTITION OF ROAD TANK SUBJECTED TO EXTERNAL PRESSURE

Petr PAŠČENKO

The thin-walled spherical cap loaded by an external pressure shows a non-linear behavior. This phenomenon is further amplified in the case of the simply supported cap with free radial displacements of the edge. Using normal calculation procedures based on the linear mechanics is not possible here. A detail numerical computational analysis using, for example, the finite element method (FEM) is necessary. In addition to large displacements, non-linear behavior of material is possible to consider. It should also be noted that stability carrying capacity might be considerably influenced by initial imperfections. Using standard procedures for calculations of real spherical caps is possible, however, often quite conservative. The question of the initial imperfection effect on stability of spherical caps is a subject of further research.

Zusammenfassung

DAS DIAPHRAGMA DER KUGELHAUBE DES TANKWAGENS, DER MIT DEM DRUCK VON AUßEN BELASTET IST

Petr PAŠČENKO

Dünnwandige sphärische Haube, die von aussem Druck geladen ist, zeigt eine nichtlineare Verhalten. Dieses Phänomen wird noch im Falle der einfach getragenen Haube für den freien Verkehr von Kanten in radialer Richtung noch verstärkt. Hier ist schon keine Möglichkeit für die Berechnung von grenzen Drucke die normallen Berechnungsmetoden auf den Grund der Linearmechanik ausnutzen. Es ist notwendig eine detaillierte numerische Analyse mit Hilfe eines Computers, wie z. B. die Finite-Elemente-Methode (FEM) durchführen. Zusätzlich zu großen Verschiebungen können die das nichtlineare Verhalten des Materials. Mehr ist zu bemerken, dass die Tragfähigkeit der Stabilität werden können, stark von der ersten Defekte. Verwendung von Standard-Verfahren für die Berechnung der fairen Ball Flügel ist möglich, aber oft sehr konservativ. Die Frage der Auswirkungen der ersten Defekte auf der Stabilität der Kugelhaube ist darum ein Gegenstand der weiteren Forschung.