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THEORETICAL BACKGROUND FOR TYRES SLIP ANGLES RESEARCH

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1. Introduction

During vehicle ride tyre properties play an important role especially concerning vehicle directional stability. Tyre slip angles are one of the most important ones and have strong influence on resulting behaviour of vehicles. Tyre slip angle appears when a rolling pneumatic tyre is subject to a lateral force. That is why this phenomenon originates just in driving in a curve when also the importance of directional stability increases. Theory supporting principle and values of slip angles has already been known for several decades. But significant progress especially in tyres design has been made since the time the theory originated. A question arises, how much the changes in tyres properties are reflected in validity of the theory solving slip angles values. The question is also connected with the increasing importance of mathematical simulations for which the level of elaboration of tyre models plays one of the key roles. It is clear that execution of a research of real values of slip angles in dependence on type and design of modern tyres would have great contribution, especially for verification of mathematical models of tyres. Experimental verification must be integral part of that research. Just in the case of tyre slip angles the task is not easy.

2. Tyre slip angles

Tyre slip angle is defined as an angle δ formed by the vector of tyre travel speed and the plane of tyre rotation.

The principle of slip angle origin results from essential characteristics of tyres, from their lateral and cornering stiffness. When a rolling pneumatic tyre is subject to a lateral force, its body plies and tread are being distorted. In consequence of the deformations the vector of the tyre real motion diverts from the plane of tyre rotation. The direction of the vector of the tyre real motion is parallel to the tangent line to the distorted tread from the point of the first contact between the tread and a road surface (see ***Fig. 1***).

With regard to slip characteristics of common tyres it is possible to consider the dependence of lateral force and slip angle to be linear and express it as:

$$Y = k \cdot \delta \qquad (1)$$

where:

Y .... lateral force [N],

k .... tyre cornering stiffness [N rad^{-1}],

δ .... slip angle [rad].

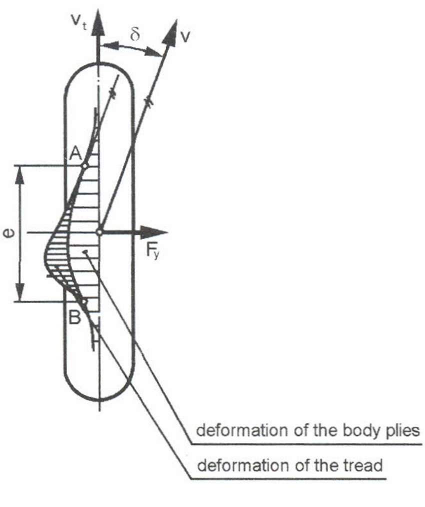

Fig. 1 *Tyre slip angle*

Slip angles contribute substantially to vehicle behaviour from the turn-in ability point of view and influence also directional stability.

To explain the importance of slip angles the simplified model of a vehicle (so called single track model) is very useful. The principle of change in vehicle behaviour during cornering consists in a change of position of centre of turning O_S and so a change of turning radius R_S (see ***Fig. 2***).

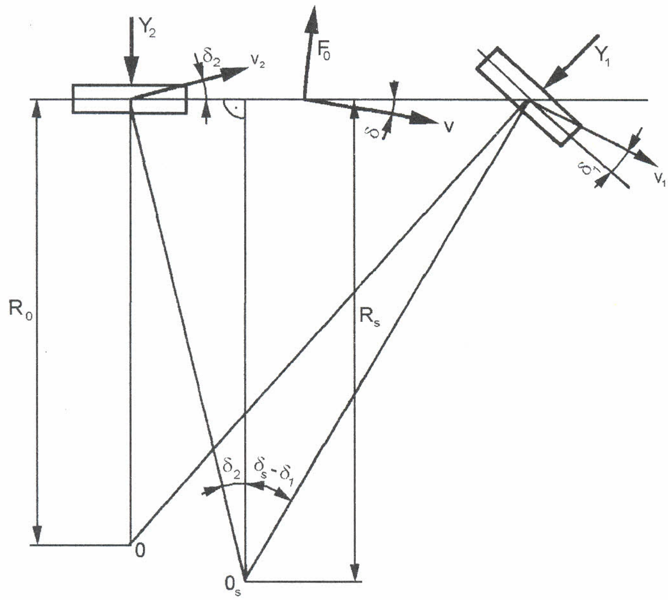

Fig. 2 *Change of position of centre of turning and value of turning radius due to slip angles*

3. Slip angles determination

Considering such a driving condition when the formula (1) is valid, then the slip angle value is directly proportional to an acting lateral force and a given tyre cornering stiffness. But two problems appear then: To what edge conditions especially given by tyre construction and driving speed is the dependence (1) valid (or linear)? And the other problem is how to determine tyre cornering stiffness. Despite of the fact during a real vehicle ride we cannot speak about tyre cornering stiffness, but cornering stiffness of whole axle, including not only tyre stiffness, but also wheel suspension stiffness, steering mechanism stiffness, clearances, etc.

So it is clear completion of series of experimental measurements is necessary to determine real dependences of slip angles and many other design and operational factors. Not until having data from the experiments it will be possible to verify or amend actual theoretical knowledge considering the current development of modern automotive tyres.

3.1 Slip angles comparison

To illustrate difficulty of the problem a comparison of slip angles acquired with various theories reflecting different approaches has been made.

Parameters of the vehicle model used for comparison are shown in the table ***Tab. 1***.

Tab. 1 *Parameters of the vehicle model*

Parameter	Symbol	Value	Unit
Weight	m	10000	kg
Wheel base	L	5	m
Track width	B	2	m
C.G. to front axle	l_1	2,5	m
Tyre dimensions		285/70 R19,5	
Tyre inflation	p	0,55	MPa

Steady-state cornering (radius 20 m, vehicle speed 30 kph) has been chosen as the model situation.

3.1.1 Theoretical computation

Firstly a computation of slip angle value on one tyre of rear axle was made. Tyre cornering stiffness was determined according to the empirical formula (2):

$$k = 0{,}5b \cdot (D + 2b) \cdot (0{,}1 + p) \text{ [kN/rad]}, \tag{2}$$

where:

b..tyre width [in],
D...rim diameter [in],
p..tyre inflation [MPa].

Then slip angle of one tyre was computed for the specified imput values according to formula (1):

$$\delta_2 = 3{,}02°.$$

3.1.2 Computation by means of tyre mathematical model

For the comparison purpose the mathematical model of a tyre from the MSA software (Multibody System Analysis) used for simulations of vehicle dynamics has been chosen. Even this model is based on the hypothesis slip characteristic is linear till the reach of adhesive limit (***Fig. 3***).

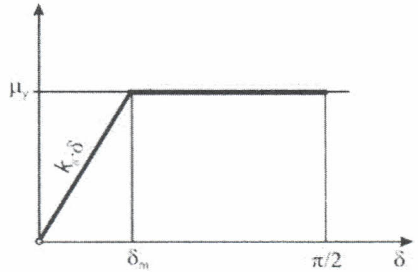

Fig. 3 *Tyre slip characteristic*

Slip angle values is possible to get as a list directly from the model used for simulation. A time behaviour of slip angle of rear axle of the model during steady-state cornering is on the picture ***Fig. 4***.

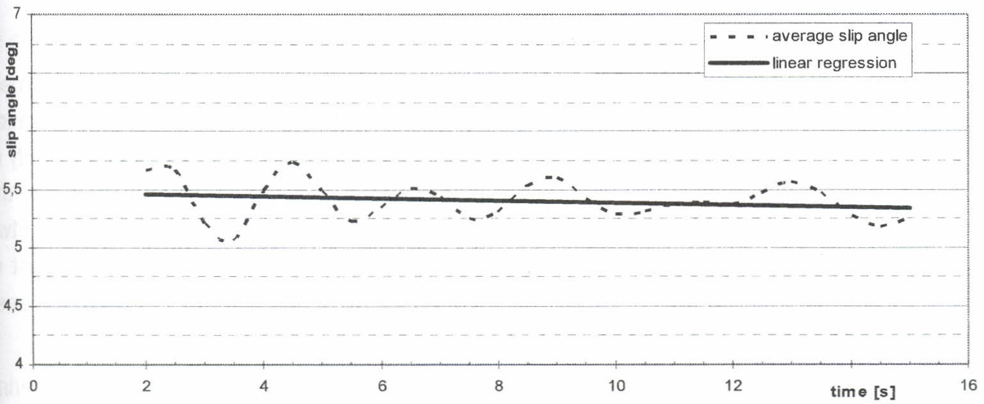

Fig. 4 *Slip angle of the rear axle (average of left and right wheel) from the simulation*

The value of slip angle of one tyre of rear axle (the average of left and right wheel) determined by simulation is:

$$\delta_2 = 5{,}4°.$$

This value is different from the value from the theoretical computation. So it is clear the experimental measurement is inevitable to find real values of slip angles.

4. Experimental methods of slip angles determination

As it was said in the chapter 3, to find real slip angle characteristics experimental methods are inevitable. The piece of knowledge we have gathered during years of our former works and multinational endeavour to improve road safety were strong motivation to continue our research in this field. The department of transport means and diagnostics has been solving these problems concurrently by means of mathematical models and driving tests of common road vehicles till now. The present state of knowledge projected into abilities of mathematical models and driving tests carried out with common vehicles are becoming limiting factors to find new characteristics or to verify new ideas. That is why a decision was taken to develop and build an experimental 2-axle vehicle. The design of the vehicle enables not only changes in vertical load and its distribution to axles (i.e. vehicle weight and position of centre of gravity) in wide range but also changes in drive and steer configuration (i.e. front-wheel drive, rear-wheel drive or four-wheel drive and front-wheel steering or four-wheel steering). The vehicle will be used for stability and handling tests, especially for experimental determination of relations between slip angles and various operating factors. Creation of an algorithm for rear axle steering improving directional stability should be the final stage of the research.

4.1 Measurement methods

To reach the goals mentioned above we are going to use our MSZ measurement system. Besides traditional quantities (vehicle speed, vehicle yaw rate, side acceleration or roll/pitch angle) slip angles of axles and wheels, speeds of various points (especially of centre of gravity) and vehicle trajectory will be determined. A detailed description of the MSZ is not interesting for this article, except sensors. The most important sensors are two optical Correvit S-CE sensors with the VG910S gyroscope for direct measurement of advance motion speed vector and yaw rate. The experimental vehicle is ready for easy installation of MSZ.

There are two methods of determining slip angles and trajectory with MSZ differing in a way of yaw rate acquisition. Both methods consider the vehicle motion as a plane motion and are based on the substitution of a rotary motion around a moving instantaneous centre of turning for the general plane motion and were first outlined in [2].

4.2 Coordinate systems

Two coordinate systems are standardized for vehicle dynamics research: the fixed coordinate system of XYZ and the vehicle coordinate system of xyz. The XYZ system is fixed to the road surface. The xyz system originates at the vehicle centre of gravity, travels with the vehicle and its x axis (longitudinal) is heading forward. The y axis (transverse) is heading left.

In order to measure slip angles with an indirect method it is useful to define an auxiliary coordinate system. The auxiliary system of x'y'z' travels with the vehicle too. Its axes of x' and y' are projections of the x and y axes to the XY plane.

4.3 Instantaneous centre of turning

The sequence for the determination of the instantaneous centre of turning (the pole) depends on the way of yaw rate acquisition.

- **Method with direct measurement of yaw rate**

A method based on direct measurement of yaw rate and advance motion speed vector of a point of a vehicle will be preferred. Installation of only one multifunction sensor (Correvit S-CE sensor with VG910S gyroscope) on a vehicle is needed. The sensor measures direction and magnitude of the speed vector (the point of action is the sensor objective) and magnitude of yaw rate.

For the sensor speed v_{C1} and the speed v_{ji} of investigated j-th point of the vehicle (e.g. the centre of unsteered axle v_{Nz}) undermentioned vector products are valid:

$$\vec{v}_{C1i} = \vec{\omega}_{C1i} \times \vec{r}_{C1i}, \tag{3}$$

$$\vec{v}_{ji} = \vec{\omega}_{C1i} \times \vec{r}_{ji} \tag{4}$$

where:

$\vec{v}_{C1i}$ .... the vector of the advanced motion speed of the Correvit sensor (measured directly),

$\vec{\omega}_{C1i}$ .... the vector of the vehicle yaw velocity (measured directly),

$\vec{r}_{C1i}$ .... the sensor position vector, the magnitude of the vector defined as on the Fig. 5 represents just the instantaneous radius of turning),

$\vec{r}_{ji}$ .... the position vector of the investigated j-th point of the vehicle,

i .... index which indicates a quantity in the instant of time of $t_i = {}^{i}/_{f_{vz}}$, where f_{vz} is the sampling rate and i = 0, 1, ... N. (N+1) is total number of samples.

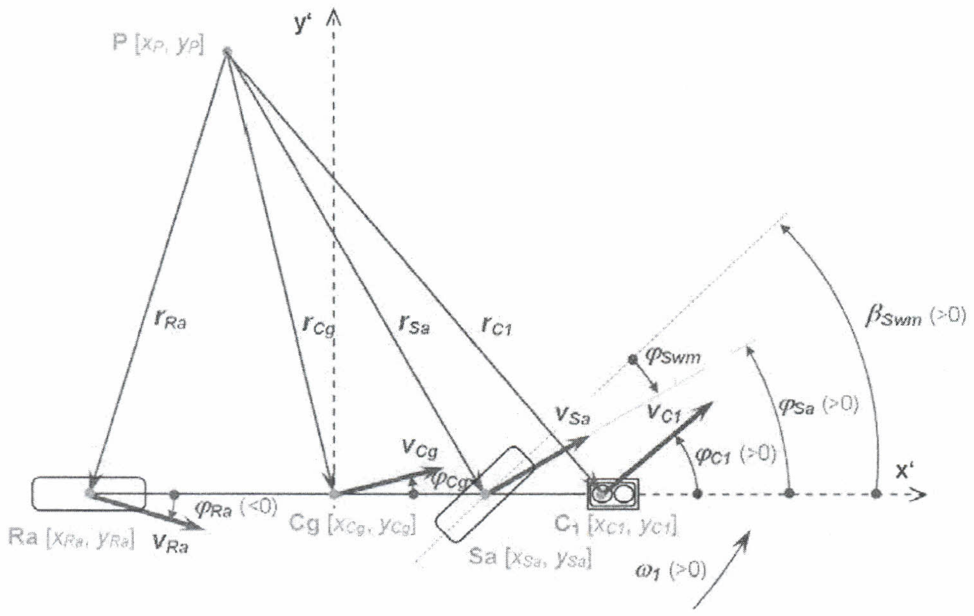

Fig. 5 *The single-track model of vehicle turning*
(C_1...the Correvit sensor, S_a/R_a...the centre of the steered/rear axle,
P... the instantaneous centre of turning, T...the centre of gravity of the vehicle)

The coordinates of the instantaneous centre of turning x_P and y_P (5) a (6) in the *x'y'z'* frame can be obtained after execution of the vector product (1) comparing the components of the resultant vector product with the sensor velocities parallel to the *x'* and *y'* axes:

$$x_{Pi} = \frac{x_{C1} \cdot \omega_{C1i} - v_{C1i} \cdot \sin \varphi_{C1i}}{\omega_{C1i}}, \tag{5}$$

$$y_{Pi} = \frac{y_{C1} \cdot \omega_{C1i} + v_{C1i} \cdot \cos \varphi_{C1i}}{\omega_{C1i}}, \tag{6}$$

where:

φ_{1i}......................... the magnitude of the slip angle of the Correvit sensor (measured directly by the sensor),

x_{C1}, y_{C1}.................. the coordinates of the Correvit sensor in the *x'y'z'* frame (instead, the coordinates of the sensor in the vehicle frame of *xyz* can be used in praxis).

- **Method with indirect measurement of yaw rate**

If there is no possibility to use a gyroscope, two sensors of the vector of the advanced motion speed can be used to determine the position of the instantaneous centre of turning. The sensors will be attached to the vehicle at different places (as far as possible from each other).

Then the coordinates the centre of turning will be determined as an intersection of pole lines of the two sensors according to (7) and (8).

$$x_{Pi} = \frac{(y_{C1} - y_{C2}) \cdot tg\varphi_{C1i} \cdot tg\varphi_{C2i} + x_{C1} \cdot tg\varphi_{C2i} - x_{C2} \cdot tg\varphi_{C1i}}{tg\varphi_{C2i} - tg\varphi_{C1i}}, \tag{7}$$

$$y_{Pi} = \frac{y_{C2} \cdot tg\varphi_{C2i} - y_{C1} \cdot tg\varphi_{C1i} + x_{C2} - x_{C1}}{tg\varphi_{C2i} - tg\varphi_{C1i}} \tag{8}$$

where:

φ_{Cji} .... magnitude of the slip angle of the speed vector of the *j*-th sensor at the instant of time t_i,

x_{Cj}, y_{Cj} .... coordinates of the speed vector sensors in the x'y'z' frame (instead, the coordinates of the sensors in the vehicle frame of xyz can be used in praxis).

4.4 Slip angles

Having the coordinates of the centre of turning (5) and (6) or (7) and (8) it is easy to compute the angular coefficient k_{Pji} of the pole line formed by the instantaneous centre of turning and the investigated *j*-th point of the vehicle with its coordinates $[x_j, y_j]$. Because the speed vector is perpendicular to the pole line, its angular coefficient equals to negative reciprocal value of the angular coefficient of the pole line. Finally the slip angle φ_{ji} will be calculated as inverse tangent function from the angular coefficient of the speed vector of the *j*-th point according to (9).

$$\varphi_{ji} = arctg\left(\frac{-1}{k_{Pji}}\right) = arctg\left(\frac{x_j - x_{Pi}}{y_{Pi} - y_j}\right), \tag{9}$$

The formula (9) will be used most often for determination of the slip angles of the vehicle centre of gravity and the centre of unsteered axle or individual wheels of unsteered axle.

Attention must be paid to the fact that the slip angle calculated according to (9) is related to the *x* axis of the vehicle coordinate system and includes steering angle or slip angle caused by suspension compliance. While the influence of compliances is all right from the vehicle point of view, the slip angle value with included steer angle is not useful. Thus it is necessary to measure wheel steer angles and to subtract their values β_{swji}. To evaluate slip angle of steered wheels φ_{swji} the formula (10) must be used (φ_{wji}.computed by analogy with (9), index *w* is used only to emphasize relation to wheel).

$$\varphi_{swji} = \varphi_{wji} - \beta_{swji}, \tag{10}$$

If slip angle φ_{Swmji} of the "middle wheel" of steered axle (in single track model) is computed, then it is possible to use the mean value β_{Swmji} of steer angles of both wheels (see ***Fig. 5***; φ_{Saji}.computed by analogy with (9), index *Sa* is used only to emphasize relation to steered axle; attention: the *j*-th points in the case (10) and (11) are different!):

$$\varphi_{Swmji} = \varphi_{Saji} - \beta_{Swmji} \tag{11}$$

4.5 Magnitudes of Speed Vectors

In order to determine the magnitude of the advance speed vector of the *j*-th inspected point the instantaneous radius of turning r_{ji} must be known. The radius can be calculated according to the formula (12).

$$r_{ji} = \sqrt{(x_{ji} - x_{Pi})^2 + (y_{ji} - y_{Pi})^2} \tag{12}$$

Because the position vectors were defined as perpendicular to vectors of advance motion speeds in this article, a scalar product can substitute for the vector product (4) and the wanted magnitude of the advance speed vector can be acquired according to the formula (13).

$$v_{ji} = \omega_{C1i} \cdot r_{ji} \tag{13}$$

The speed of the vehicle centre of gravity will be determined according to the formula (12) most often.

4.6 Yaw rate

To reconstruct the trajectory of the vehicle yaw rate must be expressed in numbers. This paragraph is irrelevant when a gyroscope is used. In the case that two speed vector sensors are used instead of a gyroscope, yaw rate can be calculated according to (14) for the first Correvit sensor and (15) for the other one.

$$\omega_{C1i} = \frac{v_{C1i} \cdot \cos\varphi_{C1i}}{y_{Pi} - y_{C1i}}, \tag{14}$$

$$\omega_{C2} = \frac{v_{C2i} \cdot \cos\varphi_{C2i}}{y_{Pi} - y_{C2i}}, \tag{15}$$

where:

v_{C1i}, v_{C2i}................. magnitudes of the vectors of the advance motion speed of the Correvit sensors

Both values (14) and (15) should be equal, but in practice small deviations can appear. An average (16) of the values (14) and (15) is used to get as close as possible to the most probable real value and the value (16) is also used for yaw angle computation (17).

$$\omega_i = \frac{\omega_{C1i} + \omega_{C2i}}{2} \tag{16}$$

4.7 Yaw angle

Vehicle yaw angle computation must precede the trajectory reconstruction too. Vehicle yaw angle is the angle formed by the x axis of the vehicle coordinate system and the X axis of fixed coordinate system. The formula (17) for the yaw angle Ω_i in the instant of time t_i follows:

$$\Omega_i = \Omega_{i-1} + T \cdot \omega_{C1(i-1)}, \tag{17}$$

where:

T... period of sampling,

Ω_0... initial value (in the instant of time t_0 = 0 s) usually equals null.

4.8 Trajectory reconstruction

Knowing all the quantities mentioned above the trajectory of any point of the vehicle can be computed according to formulas (18) for its X coordinate and (19) for its Y coordinate:

$$X_{ji} = X_{j(i-1)} + T \cdot v_{ji} \cdot \cos(\varphi_{ji} + \Omega_i), \tag{18}$$

$$Y_{ji} = Y_{j(i-1)} + T \cdot v_{ji} \cdot \sin(\varphi_{ji} + \Omega_i), \tag{19}$$

where:

$X_{j(i-1)}$, $Y_{j(i-1)}$, ... trajectory coordinates of the j-th point in the fixed coordinate system in the previous instant of time $t_{(i-1)}$; initial values X_{j0}, Y_{j0} (in the instant of time t_0 = 0 s) usually equals null,

v_{ji}, ... speed vector magnitude of the j-th point measured directly (v_{C1i} for sensor trajectory reconstruction) or calculated according to (13) (e.g. speed of the centre of gravity),

φ_{ji}, ... slip angle of the j-th point measured directly (φ_{C1i} for sensor trajectory reconstruction) or calculated according to (9) (e.g. slip angle of the centre of gravity),

Ω_i .... yaw angle acquired according to (17).

In practice the formulas (17) and (18) will be used for reconstruction of trajectory

The inspected points their trajectory will be computed most often will be the sensor objective with local coordinates of [x_{Cj}, y_{Cj}] and vehicle centre of gravity [x_T, y_T] (x_T = 0, y_T = 0).

5. Conclusion

Problems of tyres slip angles are very complex and complicated. Verification of traditional („historical") theories of slip angles or their amendment with regard to the design of contemporary tyres would be a great contribution especially for improvement of tyre mathematical models used for vehicle stability research (e.g. directional stability, turn-in ability and aerodynamic stability research as well) and subsequently for another improvement of road vehicles stability.

Experimental methods proposed and described in this article should help the department of transport means and diagnostics to participate successfully in solving the problems.

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Resumé

TEORETICKÉ PODKLADY PRO VÝZKUM SMĚROVÝCH ÚCHYLEK PNEUMATIK

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Článek popisuje problematiku směrových úchylek pneumatik a podává návrh pro jejich experimentální zjišťování. V úvodu je popsán princip vzniku směrových úchylek a jejich vliv na směrovou stabilitu a zatáčivost. Složitost problematiky je ilustrována na modelové situaci, k jejíž řešení byly použity dva různé matematické přístupy. Hlavní část článku však tvoří teoretický rozbor zatáčení vozidel za účelem získání směrových úchylek z experimentálně zjištěných dat. Popsané teoretické podklady budou využity při experimentech se speciálním dvounápravovým experimentálním vozidlem, které vzniká na katedře dopravních prostředků a diagnostiky. Vozidlo má variabilní pohon všech kol a řízením všech kol. Experimentální vozidlo a uvedené teoretické podklady mají napomoci buďto verifikaci tradičních teorií popisujících principy vzniku směrových úchylek nebo jejich modernizaci tak, aby odpovídaly vlastnostem současných pneumatik. Tyto teorie pak budou použity pro návrh algoritmu pro řízení zadní nápravy za účelem zvýšení směrové stability vozidla.

Summary

THEORETICAL BACKGROUND FOR TYRES SLIP ANGLES RESEARCH

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The article deals with problems of tyre slip angles and gives a proposal for their experimental determination. It describes the principle of slip angles origin and their influence on directional stability of road vehicles. The complexity of their mathematical determination is ilustrated on a model situation with utilization of two different theoretical approaches. But the main part of the article is a theoretical analysis of vehicle turning for the purpose of slip angles determination from experimentally acquired data. The theoretical background described in this article will be used for slip angles evaluation in experiments carried out with an experimental 2-axle vehicle. The vehicle has been designed at the department of transport means and diagnostics and has a variable all-wheel drive and all-wheel steering. The experimental vehicle and this theoretical background should help either to verify the traditional theories supporting principle of slip angles or to ammend them to correspond to type and design of modern tyres. As the final stage of the research the verified or ammended theories will be used for creation of an algorithm for rear axle steering in order to improve directional stability.

Zusammenfassung

THEORETISCHER HINTERGRUND FÜR SCHRÄGLAUFWINKELS FORSCHUNG

Tomáš ZIKMUND, Miroslav TESAŘ

Der Artikel beschreibt Probleme der Schräglaufwinkel von Reifen und gibt einen Vorschlag für ihre experimentelle Bestimmung. Der größte Teil des Artikels ist eine theoretische Analyse der Kurvenfahrt des Fahrzeugs für die Zwecke der Bestimmung der Schräglaufwinkels aus experimentell gewonnenen Daten.