ELECTRO ACOUSTIC ANALOGIES IN MATLAB ENVIRONMENT

Jiří Kulička[1]

Analogy is a cognitive process of transferring information from a specific object (source) to another specific object (goal), in this case means a mathematical similarity between different physical systems and processes. This article deals with the method of calculating the resonant frequency acoustic system on the basis of electro acoustic analogy, based on isomorphism various physical systems - electrical, mechanical, hydraulic and electro acoustic. Acoustic systems can be created by combining their individual components, like electrical circuits or mechanical and hydraulic systems. The acoustic resonance in the system always takes place when it has alternating power and its impact coincides with the natural frequency of vibration.

The article created electro acoustic model of flute, which is studied using a computer model in an environment of Matlab and Simulink. The model works by calculating the acoustic impedance, which is determined by the resonant frequencies. Resonance occurs always in zero point graph of a function, the impedance changes its value from negative to positive.

This model leads to very decent results when we compared the theoretical values of resonant frequencies with calculated. The possibility of experimental investigation of acoustic systems, the equivalent electrical circuit is in economic terms, time and according to the results and accuracy of better solutions than the investigation of their own systems. The model can be used to design dimensions of the flutes and other acoustic systems, or to improve their debugging.

Key words: auditory system, resonant frequency, electro acoustic analogy, electro acoustic model, flute, Matlab, acoustic impedance

1 Introduction

Currently requirements for quality of acoustic systems, musical instruments and sound devices grow. Their issues are very broad and complex. This model also has quite complex theoretical basis.

The aim of my paper is to create a computer model of the acoustic system to determine its resonant frequency according to measured parameters.

Electro acoustic analogies are the basis of determining of resonant frequencies method. On the basis of the acoustic and electrical engineering, I have created an electro acoustic model flute which is studied in environment of Matlab.

[1] Mgr. Jiří Kulička, University of Pardubice, Jan Perner Transport Faculty, Department of Informatics in Transport, Studentská 95, 532 10 Pardubice, Czech Republic, tel.: +420 466 036 428, E-mail: jiri.kulicka@upce.cz

2 Acoustics scheme and electro acoustic analogy

Using the electro acoustic analogy scheme of acoustic can be converted to an equivalent electrical circuit. Part *A* – cavity of head of flute – equivalent to acoustical capacitance, part *B* – body of flute – equivalent to acoustic and part *C* – opening the mouthpiece – equivalent to acoustical mass. Their impedances can be marked Z_A, Z_B and Z_C, as shown in figures 1 and 2. Z_1 is the impedance of the end of the whole flute.

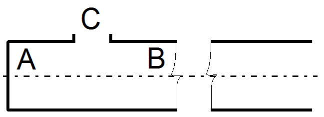


Figure 2

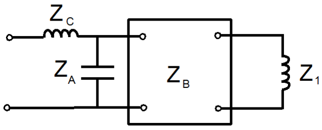


Figure 3

The resulting impedance *Z* is calculated as shown in Figure 2:

$$Z = Z_C + \frac{Z_A Z_B}{Z_A + Z_B}. \tag{1}$$

Oscillations of flute are controlled by open–speed valve. Acoustic impedance is defined as a ratio of pressure and velocity, thus resonance occurs at maximum speed, i.e. with minimal impedance. To simplify we are considering that the total impedance is equal to zero [2].

3 Impedance of parts of acoustic system

Impedances as complex numbers will be marked with capital letters and small ones will refer to their imaginary part.

3.1 Impedance of cavity of head – Z_A

Cavity of head of flute is an acoustic capacitance. To calculate the acoustic capacitance *C*, according to [6] is set equation:

$$C = \frac{V}{\rho c^2} = \frac{\pi R^2 p_0}{\rho c^2}. \tag{2}$$

Where *V* is in this case capacity of head of flute and it is designed by formula: $V=\pi R^2 p_0$, *R* is radius of the body of flute, p_0 is head length of flute, ρ is air density and *c* is speed of sound.

Acoustic capacitance corresponds to analogies of electro capacity. Its impedance is calculated:

$$Z_A = \frac{1}{i\omega C} = \frac{\rho c^2}{i\omega S p_0} = -i\frac{\rho c^2}{2\pi^2 R^2 f p_0} = -iz_A. \tag{3}$$

Where $\omega = 2\pi f$ is a circular frequency, *f* is a frequency and *i* is a complete unit.

3.2 Impedance of opening mouthpiece

Opening of mouthpiece is an acoustic mass. To calculate acoustic mass *M* according to [4] is set equation:

$$M = \frac{\rho\left(d_{15} + \frac{4}{3}r_{15}\right)}{\pi r_{15}^2}. \tag{4}$$

Where ρ is air density, d_{15} is thickness of mouthpiece, r_{15} is radius of a hole of mouthpiece see Figure 3 and diagram A.

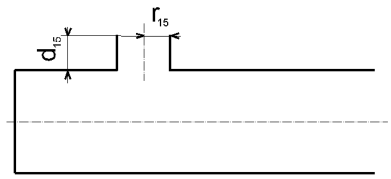


Figure 4

Acoustic mass corresponds to electro acoustic analogies of electro inductance. Its impedance is calculated:

$$Z_c = i\omega M = i\omega\frac{\rho\left(d_{15} + \frac{4}{3}r_{15}\right)}{\pi r_{15}^2} = i\frac{2f\rho\left(d_{15} + \frac{4}{3}r_{15}\right)}{r_{15}^2} = iz_C. \tag{5}$$

3.3 *Impedance of end-hole off lute – Z_1*

According to [4], impedance of the end of tube is equal to:

$$Z_1 = \frac{\rho c}{S}k\xi Ri \text{, after treatment } Z_1 = 1{,}26\frac{\rho f}{R}i = b_1 i = B_1. \tag{6}$$

Where ρ is air density, c is speed of sound, $k = \frac{\omega}{c}$ is wave constant, ξ is constant and according to [3] is its value for open tube with no outer edge is equal to 0,63.

3.4 *Determination of impedance of tube using terminal impedance*

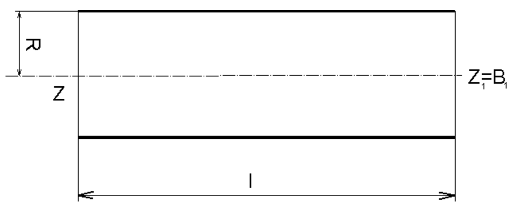


Figure 5

Resulting impedance of tube according to figure 4, according to [7, 8] is equal to:

$Z=\frac{\rho c}{S}\frac{Z_1 S\cos kl+i\rho c\sin kl}{\rho c\cos kl+iZ_1\sin kl}$, after treatment, provided that $kl \neq \pi / 2 + a\pi$, where a comes through all integers, is

$$Z=i\frac{b_1+\frac{\rho c}{S}tgkl}{1-b_1\frac{S}{\rho c}tgkl}. \tag{7}$$

Where ρ is air density, c is speed of sound, $k=\frac{\omega}{c}$ is wave constant, S is end-hole of flute area, Z_1 is end–hole of flute impedance and b_1 is imaginary part of end impedance.

4 Impedance of body of flute – Z_B

To determine the impedance of body of flute I must identify the sequence of flute impedances $\{B_j\}_{j=1}^{n}$, where n denotes the number of open holes in various tones of touch. The first member of the sequence is determined by the impedance of the end of hole derived in part 3.3. To determine the next member I must always determine even sequences $\{M_j\}_{j=1}^{n}$, sequence impedances of open holes and $\{Z_j\}_{j=1}^{n}$, which is a sequence of impedances between particular open holes. The member of the sequence B_N get passing parallel impedances of M_N and Z_N.

The calculation procedure is possible to observe in Chart A and Figure 5.

The first step - all openings closed:

$$B_1=Z_1=b_1 i$$

The second step - one hole opened:

$$M_2=i\omega\frac{\rho\left(d_2+\frac{4}{3}r_2\right)}{\pi r_2^2}=im_2 \quad Z_2=i\frac{b_1+\frac{\rho c}{\pi R^2}tgkl_2}{1-\frac{b_1\pi R^2}{\rho c}tgkl_2}=iz_2$$

$$B_2=\frac{Z_2M_2}{Z_2+M_2}=i\frac{z_2m_2}{z_2+m_2}=ib_2$$

N-th step - n-1 holes opened, see Figure 5 and Chart A:

$$M_n=i\omega\frac{\rho\left(d_n+\frac{4}{3}r_n\right)}{\pi r_n^2}=im_n \quad Z_n=i\frac{b_{n-1}+\frac{\rho c}{\pi R^2}tgkl_n}{1-\frac{b_{n-1}\pi R^2}{\rho c}tgkl_n}=iz_n$$

$$B_n=\frac{Z_nM_n}{Z_n+M_n}=i\frac{z_nm_n}{z_n+m_n}=ib_n$$

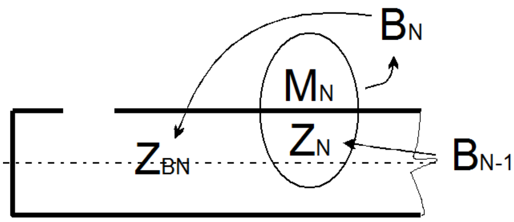


Figure 6

This determines the sequence of impedances $\{B_j\}_{j=1}^{n}$ by its n-th member. I can calculate impedance Z_{BN} by substituting into (7) for $b_1 = b_{N-1}$. For n opened holes is equal to:

$$Z_{BN} = i\frac{b_{N-1} + \dfrac{\rho c}{\pi R^2} tg\left[k\left(l_{15} + \sum_{j=N+1}^{14} l_j\right)\right]}{1 - b_{N-1}\dfrac{\pi R^2}{\rho c} tg\left[k\left(l_{15} + \sum_{j=N+1}^{14} l_j\right)\right]} = iz_{BN}\,. \tag{8}$$

Where ρ is air density, c is speed of sound, $k = \dfrac{\omega}{c}$ is wave constant, R is radius of flute, i is a complex unit and b_N is an imaginary part of impedance of sequence $\{B_j\}_{j=1}^{n}$.

5 Total impedance of acoustic system

Z_{BN} substituting into (1) we get:

$$Z = Z_C + \frac{Z_A Z_{BN}}{Z_A + Z_{BN}} = i\left(z_C + \frac{z_A z_{BN}}{z_A - z_{BN}}\right). \tag{9}$$

Where z_C, z_A and z_{BN} are imaginary parts of impedance of different parts of flute.

6 Field of touches of fundamental tone of flute

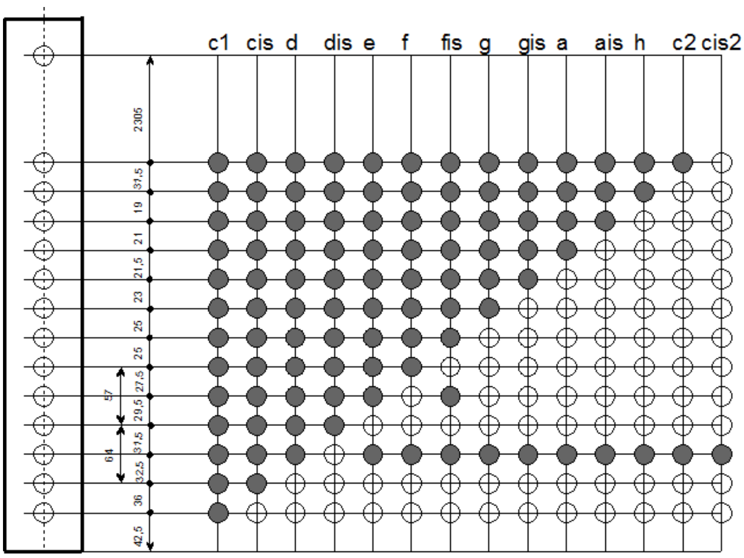


Figure 7 Touches of fundamental tone and pitch holes in mm

7 Chart A

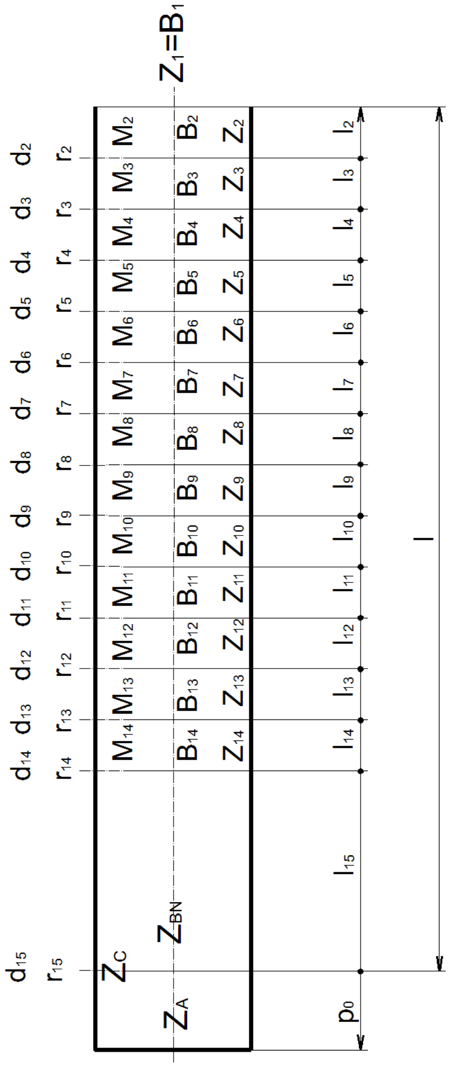


Figure 8 Constants audio system and method of calculation, $l_1 = r_1 = d_1 = 0$

8 Table dimensions

Dimensions are given in meters, marking according to chart A. Size $p_0 = 0.05$ m.

Table 3

	l	r	d
1	0	0	0
2	0,0425	0,00765	0,003
3	0,036	0,00765	0,003
4	0,0325	0,00765	0,002
5	0,0315	0,0065	0,002
6	0,0295	0,0065	0,002
7	0,0275	0,0065	0,002
8	0,025	0,0065	0,002
9	0,025	0,0065	0,002
10	0,023	0,0065	0,002
11	0,0215	0,0065	0,002
12	0,021	0,0065	0,002
13	0,019	0,0065	0,002
14	0,0315	0,0035	0,001
15	0,2305	0,0045	0,006

9 Calculations

Calculations are due complexity of program implemented in Matlab environment.

9.1 Matlab

Matlab is an integrated environment in which mathematical calculations, modelling, analysis and visualization of data, measurement, measurement and data processing, creation of algorithms, design management and communication systems and more can be performed. It contains following basic parts: computing core, graphics subsystem, working tools, toolboxes and others. The basis is computational core, performing numerical operations with matrices of real or complex numbers. Graphic subsystem allows easy viewing of the results of calculations. Working tools allow full application programming. Toolboxes are libraries of functions and they are oriented to specific disciplines.

In my work I used so-called m-files, which are actually text files that are used to write sequences of commands that can Matlab execute.

9.2 Example of m-file for tone fis

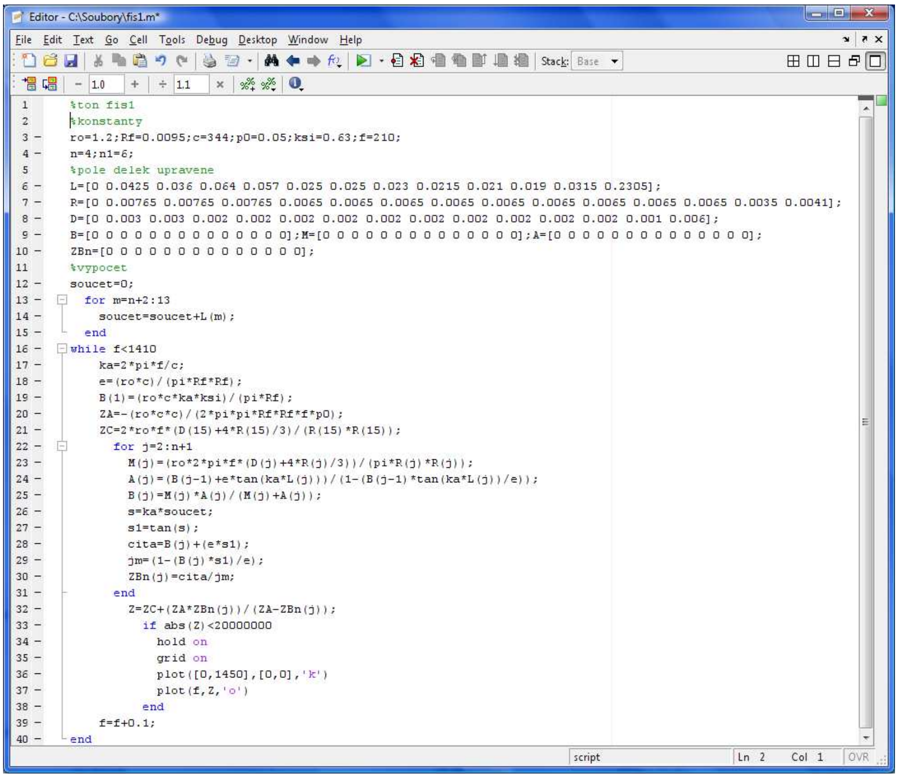


Figure 9 Calculation of impedance curve in Matlab

9.3 Example of impedance curves for tone fis1 (impedance dependence on frequency)

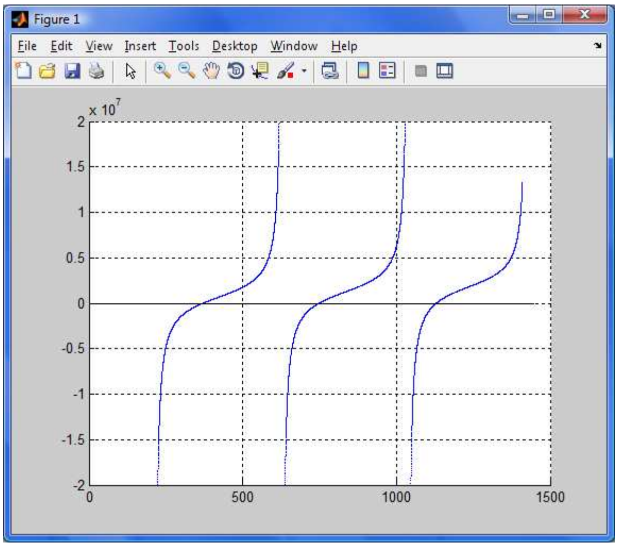


Figure 10 Impedance curve for tone fis1

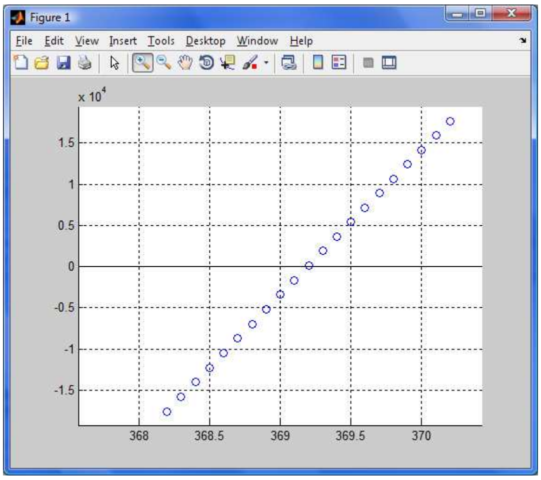


Figure 11 Detail from Figure 9, where curve intersects x-axis

9.4 Table of results

Table 4

Tone Results with l_{15}=0.2305 m	theoretical values (Hz)		calculated values (Hz)		mistakes (Hz)		deviation in cents	
	1st octave	2nd octave	1st octave	2nd octave	1st octave	2nd octave	1st octave	2nd octave
c1	261.5	523	261.4	532	-0.1	9	0	21
cis	277	554	275.9	562.6	-1.1	8.6	-5	19
d	293.5	587	293	599.2	-0.5	12.2	-2	25
dis	311	622	311.2	638.3	0.2	16.3	1	31
e	329.5	659	329.3	676.9	-0.2	17.9	-1	33
f	348.5	697	349.7	721.9	1.2	24.9	4	43
fis	370	740	369.2	763.6	-0.8	23.6	-3	38
g	392	784	393.7	819.3	1.7	35.3	5	54
gis	415	830	418.8	875.3	3.8	45.3	11	65
a	440	880	436.4	915	-3.6	35	-10	48
ais	466	932	463.4	976.2	-2.6	44.2	-7	57
h	494	988	493.2	1044.1	-0.8	56.1	-2	68

c2	523	1046	524.3	1115.8	1.3	69.8	3	80
cis2	586	1208	597.6	1282.3	11.6	74.3	24	74

We can see in Table 1 that in the first octave tones resonate quite well in line with theoretical values. Deviations of a few cents are not perceptible even by experienced hearing music. Exception is tone cis2 added higher of 24 cents. This corresponds to practice because flautists complain about this tone that it sounds higher. There should be appropriate to move this hole when manufacturing instruments.

Table 5

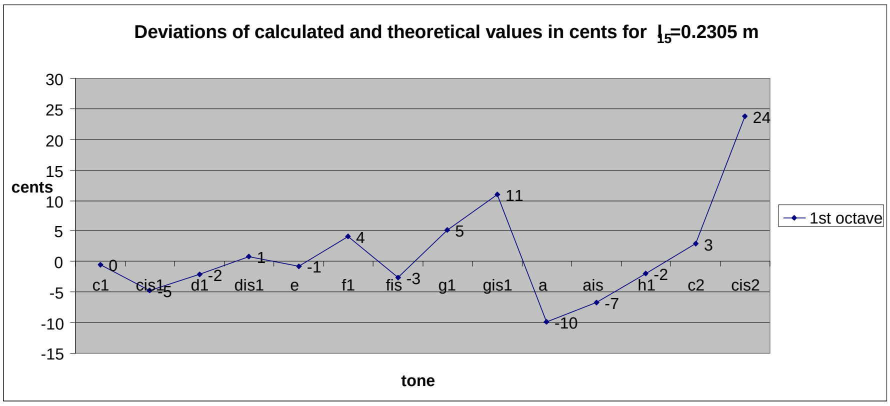
Tone Results with l_{15}=0,2355m r_{15}=0,0033 m	theoretical values (Hz)		calculated values (Hz)		mistakes (Hz)		deviation in cents	
	1st octave	2nd octave	1st octave	2nd octave	1st octave	2nd octave	1st octave	2nd octave
c1	261.5	523	-	518.8	-	-4.2	-	-10
cis	277	554	-	548.4	-	-5.6	-	-12
d	293.5	587	-	583.9	-	-3.1	-	-6
dis	311	622	-	621.9	-	-0.1	-	0
e	329.5	659	-	659.4	-	0.4	-	1
f	348.5	697	-	703	-	6	-	10
fis	370	740	-	743.6	-	3.6	-	6
g	392	784	-	797.6	-	13.6	-	21
gis	415	830	-	851.9	-	21.9	-	32
a	440	880	-	903.3	-	23.3	-	32
ais	466	932	-	964.5	-	32.5	-	42
h	494	988	-	1032.5	-	44.5	-	54
c2	523	1046	-	1104.3	-	58.3	-	67
cis2	586	1208	-	1248.8	-	40.8	-	41

Because calculated frequencies in the second octave are higher than theoretical ones, it is necessary to tune instrument by extension of a part marked as l_{15} (selected extension by 0.5 cm to 0.2355 meters). I have also chosen a smaller radius hole of mouthpiece because flautists stress chops when playing higher tones thereby they reduce area of hole (r_{15} = 0.0033 meters). Using these values I achieved acceptable results as can be seen in Table 3.

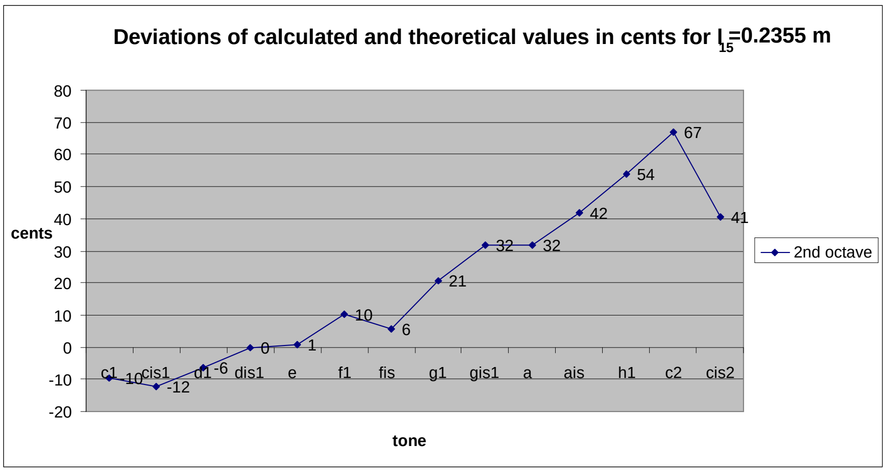
For tones from g2 higher, I proceeded one more tuning l_{15} = 0.245 m.

Tone Results with l_{15}=0,245m r_{15}=0,0033 m	theoretical values (Hz)		calculated values (Hz)		mistakes (Hz)		deviation in cents	
	1st octave	2nd octave	1st octave	2nd octave	1st octave	2nd octave	1st octave	2nd octave
g	392	784	-	778.8	-	-5.2	-	-8
gis	415	830	-	830.4	-	0.4	-	1
a	440	880	-	885.3	-	5.3	-	7
ais	466	932	-	943.9	-	11.9	-	15
h	494	988	-	1008.9	-	20.9	-	25
c2	523	1046	-	1077.2	-	31.2	-	36
cis2	586	1208	-	1199.9	-	-8.1	-	-8

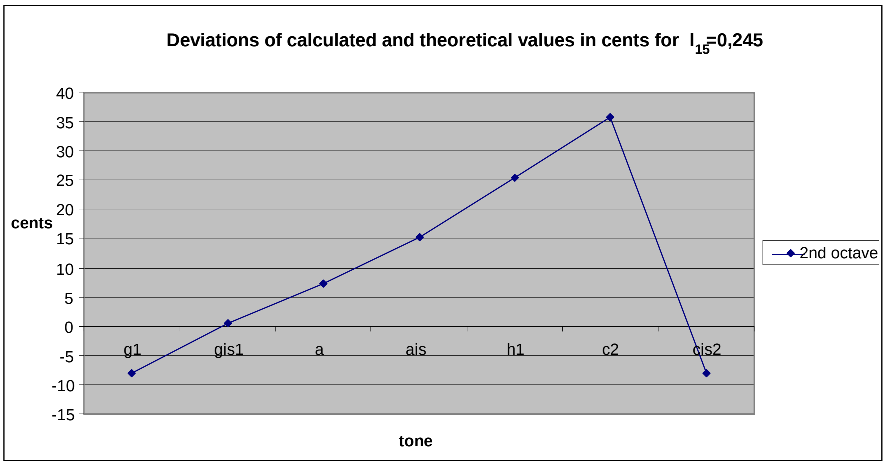
9.5 Graphs of deviations calculated from theoretical values



Graph 1



Graph 2



Graph 3

10 Conclusion

Comparing theoretical and calculated values, it is shown that this method of calculation leads to very good results. It is seen that lower tones tend to sound lower and higher tones tend to sound higher. Therefore flautists have to choose a compromise and fine-tune by ear, by relaxing or stressing their chops (increase or reduce space between lips and mouthpiece adapted to size of area). We can see that the first octave of tones agrees quite well with theoretical frequencies. Differences of a few Hz are not perceptible even by a trained ear. Tones of the second octave from g2 up sound higher and therefore a flautist either stresses his/her chops, thereby frequency of tones is reduced or it is fine-tuned by pulling head of flute, which I marked L15 and thereby instrument becomes extended, thereby frequency of tones decrease. These conclusions, however, agrees very well with practice and also with conclusions of J.W. Coltmanna who carried out experimental measurements and justified this [1].

My theory is based on fundamental relationship between acoustic units which are derived in literature by experimental way and therefore not entirely accurate. It is possible that some inaccuracies are caused by this factor. Impedance is generally a complex number but there is always based on a purely imaginary one. It is due to the fact that literature neglects real component of the end impedance [3]. If they wanted to see more accurate results, it is necessary to deal with this problem to determine precise relations for individual acoustic units, to learn what change of section and moving a hole could influence, what impact on resonant frequency acoustic friction and many other effects could have.

The work followed the use of results in practice. Further investigation of this model may help in the future to design an ideal musical instrument that has optimized dimensions so as to avoid influence of imperfections of a musician who plays it, and effects of environment where the instrument is played. The model can also be used in areas where it is necessary to design acoustic system which has desired frequency noises.

Literature Reference

1. Coltmann, J. W. Resonance and Sounding Frequencies of the flute. JASA, 1966, Pitsburgh, USA, č. 1, 99-107, ČSHN č.1311 Hradec Králové.
2. Benade, A. H. On theTone and Response of Wind Instruments from an Acoustical Standpoint. Univerzitní tisk Case Western Reserve University, Ohio, 44106, březen 1972, ČSHN č. 1976.
3. Guitard, J. Impedances terminales de Tuyaux Sonores Cylindriques, Acustica 1962, č. 5, ČSHN.

4. 1st publication. Praha: Grada, 1998. 291 pp. ISBN 80-7169-608-0.
5. Podobský J. Vyšetřování hornopropustných akustických filtrů. Hluk a otřesy, Sborník čs. vědecké technické společnosti, ČSAV, Praha 1959.
6. Podobský J. Procházky akustikou I. díl. MAFY Hradec Králové 1999. ISBN 80-86148-28-9.
7. Podobský J. Procházky akustikou II. díl. MAFY Hradec Králové 2005. ISBN 80-86148-75-0.
8. Merhaut, J. Teoretické základy elektroakustiky. Praha: ACADEMIA – ČSAV, 1976.
9. Nedervenn, C.J. Calculations on location and dimensions of holes in a clarinet. Acustica 1964 č. 4. ČSHN 891.