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CREEPING MOTION OF A SINGLE FLUID AND SOLID SPHERE IN NON-NEWTONIAN FLUIDS

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An approach to the calculation of drag and velocity of a single Newtonian fluid sphere moving slowly in generalized Newtonian and viscoplastic fluids is suggested. It is based on the application of the modified Rabinowitsch–Mooney equation together with the corresponding relations for consistency variables, which have been suggested for the creeping motion of the solid sphere, and on the application of the relationship of Hadamard–Rybczynski valid for both fluid and solid spheres. The solution is concretized for the power-law, Ellis, Bingham, and Robertson–Stiff flow models.

Introduction

The motion of droplets or gas bubbles in non-Newtonian fluids is encountered in a wide range of chemical processes. Typical examples include polymerisation processes, food processing, activated sludge process, fermentation processes, etc.

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The solution of creeping motion of a single fluid sphere represents a theoretical starting point for solving such practical problems and received much attention in the past.

The pioneering solution to the problem of the creeping motion of a Newtonian fluid sphere in Newtonian fluids (NF) is available from the work by Hadamard and Rybczynski (according to [1]). The majority of the papers dealing with the motion of a Newtonian fluid sphere in non-Newtonian fluids have been summarized by Chhabra [2]). From this review it is evident that most widely studied is the problem of slow motion of a single fluid sphere in simpler generalized Newtonian fluids (GNF), the same being true of recent papers (e.g. [3–5]). However, the problem of creeping motion of a single fluid sphere in more sophisticated viscoplastic fluids (VPF) (e.g. biological material) has attracted only little attention so far [6].

The present paper forms a continuation of our earlier papers dealing with the approximate solution to the problem of the momentum transfer in the non-Newtonian fluids-solid particle systems with the use of equations of the Rabinowitsch–Mooney type [7–10]. Its aim is to extend the region of validity of the relationship suggested for slow motion of a single solid sphere in GNF and VPF [8] also for slow motion of Newtonian fluid sphere.

Theory

Using the Stokes approach, the equation of Rabinowitsch–Mooney type (Eq. (1)) with the momentum balance in the form given by Eq. (2) has been suggested for external flow of GNF and VPF past a single solid spherical particle [8]

$$\dot{D}_{w,p} \equiv \frac{2u_{ch}}{d} = \frac{\tau_{w,p}^{1/2}}{2} \int_{\tau_0}^{\tau_{w,p}} \tau^{-3/2} \dot{D}(\tau) d\tau \qquad (1)$$

$$f_t - f_{fr} - f_s \equiv f_t - f_{fr} \Phi_p = 0 \qquad (2)$$

Here $\dot{D}_{w,p}$ and $\tau_{w,p} \equiv f_{fr}$ are the consistency variables, the characteristic velocity, u_{ch}, is the maximum velocity, d is the diameter of the particle, f_t is the total resistance, f_{fr} is the frictional resistance, f_s is the shape resistance of solid spherical particle referred to its surface area, τ_0 is the yield stress, $\dot{D}(\tau)$ is the dependence of the shear rate $\dot{D}$ on the shear stress τ, whose course is generally given by the flow curve of non-Newtonian fluid or by the respective flow model, and the quantity Φ_p, called the resistance factor, is given by the ratio of total to frictional resistance of a solid spherical particle. For $\tau_0 = 0$, Equation (1) assumes the form valid for an GNF.

The resistance factor Φ_p is given by the relationship

$$\Phi_p = 1 + \Psi_p + \left(\frac{\tau_0}{\tau_{w,p}}\right)^{1/2} \equiv 1 + \Psi_p + \xi_p^{1/2} \tag{3}$$

where the quantity Ψ_p, called the resistance number, is given by the ratio of the shape resistance of spherical particle caused by viscosity effects, to the frictional resistance. Then the term $1 + \Psi_p$ expresses the influence of viscosity effects and the quantity $\xi_p^{1/2}$ expresses the influence of plastic effects on the dimensionless value of total resistance of single particle.

If we write the Rabinowitsch–Mooney equation (1) in the form of Eq. (4), then the system of Eqs (1) and (4) also defines the relationship necessary for calculation of the effective viscosity of the system, $\mu_{e,p}$

$$\dot{D}_{w,p} = \frac{\tau_{w,p}}{\mu_{e,p}} \tag{4}$$

For an NF, $\mu_{e,p} = \mu$, where μ is the dynamic viscosity. Equations (1) – (4) only apply to an incompressible fluid (with ρ = const., where ρ is the density). Using the quantities ρ, u_{ch}, d, and $\mu_{e,p}$, we can express the Reynolds number as

$$Re_p = \frac{\rho u_{ch} d}{\mu_{e,p}} \tag{5}$$

After substituting Eq. (4) in combination with Eq. (1) into Eq. (5) we obtain the relationship

$$Re_p = \frac{2 \rho u_{ch}^2}{\tau_{w,p}} \tag{6}$$

which for an VPF can be written in the form

$$Re_p = \frac{2\rho u_{ch}^2 \xi_p}{\tau_0} \tag{7}$$

The dependence of resistance number Ψ_p on the Reynolds number Re_p has been presumed to be identical for an NF, GNF and VPF so that in the creeping region of flow the value of quantity Ψ_p is the same as in the Stokes approach for an NF ($\Psi_p = 1/2$).

For a fall and a rise of a spherical particle we get, from momentum balance (Eq. (2)), the following relation for the $\tau_{w,p} \equiv f_{fr}$ quantity

$$\tau_{w,p} \equiv \frac{f_t}{\Phi_p} = \frac{|(\rho_p - \rho)| g d}{6\Phi_p} \tag{8}$$

where ρ_p is the density of the spherical particle and g is the gravitational acceleration. In the case of the fall and the rise of the particle, the characteristic velocity u_{ch} then corresponds to the terminal velocity u_t in a wall-unbounded system.

The total resistance of a Newtonian fluid sphere (with clean interface) referred to its surface area translating with constant velocity in another incompressible immiscible NF in the creeping region of flow is given by the relationship suggested by Hadamard and Rybczynski (according to Ref. [1])

$$f_{t,f} = 3\Omega\mu_c \frac{u_{ch}}{d} \equiv \Omega f_t = \frac{2 + 3\beta}{3 + 3\beta} f_t \tag{9}$$

where Ω is the drag correction factor,

$$\beta = \frac{\mu_f}{\mu_c} \tag{10}$$

and the subscripts f and c are related to the fluid sphere and the continuous phase, respectively.

For a gas bubble rising through a fluid it is $\beta \ll 1$, and Eq. (9) predicts $\Omega = 2/3$, whereas for a solid sphere moving in a fluid it is $\beta \to \infty$, and Ω approaches unity. Thus Eq. (9) embraces the complete spectrum of particles ranging from gas bubbles to solid spheres.

In analogy to the solution for a solid sphere we presume the validity of relationships (9) and (10) also for GNF and VPF if the dynamic viscosity of continuous phase μ_c is substituted by the effective viscosity $\mu_{e,p}$ calculated from Eqs (1) and (4).

Results and Discussion

Generalized Newtonian fluids

For the power-law flow model

$$\dot{D}(\tau) = \left(\frac{\tau}{K}\right)^{1/n} \tag{11}$$

where K and n are its parameters, the solution of Eq. (1) with $\tau_0 = 0$ and of the momentum balance (Eq. (2)) with $\Phi_p = 3/2$ leads to the relationship

$$f_t = K\frac{3}{2}\left(\frac{2-n}{n}\right)^n\left(\frac{2u_{ch}}{d}\right)^n \tag{12}$$

After introducing the drag coefficient $C_D = 8f_t/(\rho u_{ch}^2)$, the Reynolds number in the form used in the literature for power-law fluid $Re_{PL} = \rho u_{ch}^{2-n} d^n/K$, and the relation resulting from Eq. (9) ($C_{D,f} = \Omega C_D$), the relationship (12) can be written in the dimensionless form

$$C_{D,f} = \frac{24\Omega}{Re_{PL}} Y_{(n)} \tag{13}$$

where the correction factor for a power-law fluid $Y_{(n)}$ is given as

$$Y_{(n)} = 2^{n-1}\left(\frac{2-n}{n}\right)^n \tag{14}$$

A large body of literature on the subject of the motion of a gas bubble ($\Omega = 2/3$) in a power-law fluid is available, which permits a comparison of the present expression with the previous ones. The correction factor for a power-law fluid recommended by Hirose and Moo–Young [11] has the form

$$Y_{(n)} = 2^{n-1}3^{(n-1)/2}\frac{13+4n-8n^2}{(2n+1)(n+2)} \tag{15}$$

Bhavaraju *et al.* [6] suggested the following relationship for the correction factor $Y_{(n)}$

$$Y_{(n)} = 2^{n-1}3^{(n-1)/2}\left[1-3.83\,(n-1)\right] \tag{16}$$

and Rodrigue *et al.* [4] recommended the relationship in the form

$$Y_{(n)} = 2^{n-1}3^{(n-1)/2}\frac{1+7n-5n^2}{n(n+2)} \tag{17}$$

Gummalam and Chhabra [12] calculated the so-called lower bound on $Y_{(n)}$. In the case of a single bubble, their expression becomes

$$Y_{(n)} = 2^{n-1}3^{-(n+1)/2}\left(\frac{(2n+1)(2-n)}{n^2}\right)^2 \tag{18}$$

Both our own solution and relationships (15) – (18) contradict the solution by Dewsbury *et al.* [13], which recommends the value of $Y_{(n)} = 1$ for slow rise of both light solid particles and gas bubbles.

Comparison of Eqs (14) – (18) with experimental results obtained for pseudoplastic fluids is presented in Fig. 1. Therefrom it is obvious that in the region of values of parameter $0.7 < n \leq 1$ the results obtained by application of our own relationship are practically identical with those obtained from relationships (15) and (17). Equations (15) – (17) are based on the linearised form of viscous term in the momentum balance and, therefore, these expressions can only be used for fluids with weak non-Newtonian effects.

One of possible reasons of the deviations of our own relationship from the majority of experimental results as well as their scattering can lie in the application of the simplest power-law flow model, which did not respect the Newtonian behaviour of GNF in the region of $\tau \to 0$. The case of a solid sphere ($\Omega = 1$) is similar, a satisfactory agreement with experimental results being only possible with application of flow models containing zero shear viscosity η_0 as one of their

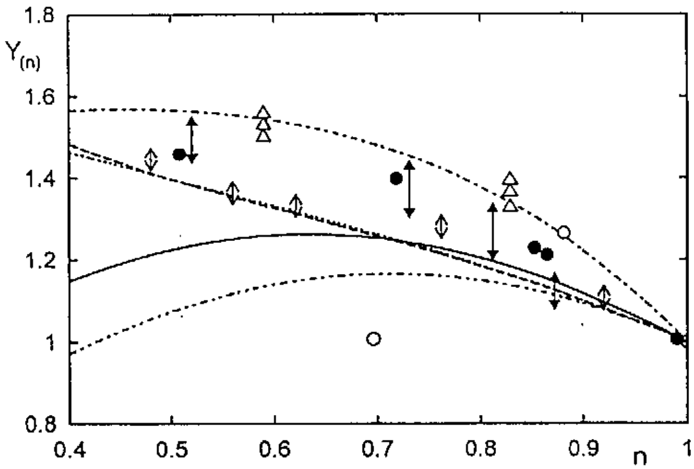

Fig. 1 Comparison of our treatment of creeping motion of gas sphere in power-law fluid with experimental data and relationships given in literature: ○ – Hirose and Moo Young [11]; ● – Acharya *et al.* [14]; △ – Bhavaraju *et al.* [6]; ↕ – Haque *et al.* [15]; ↕ – Chhabra and Bangun [5]; ——— Eq. (14); ······ Eq. (15); —·—·—· Eq. (16); – – – – Eq. (17); —··—··—·· Eq. (18)

parameters [2], [8].

For the Ellis flow model

$$\dot{D}(\tau) = \frac{\tau}{\eta_0}\left[1 + \left(\frac{\tau}{\tau_{1/2}}\right)^{\alpha-1}\right] \tag{19}$$

which respects the Newtonian behaviour of GNF in the region $\tau \to 0$ and where η_0, $\tau_{1/2}$ and α are its parameters, the solution of Eq. (1) with $\tau_0 = 0$ yields

$$\dot{D}_{w,p} = \frac{\tau_{w,p}}{\eta_0}\left[1 + \frac{1}{2\alpha - 1}\left(\frac{\tau_{w,p}}{\tau_{1/2}}\right)^{\alpha-1}\right] \tag{20}$$

The solutions given in literature for the Ellis flow model can be expressed by the relationship

$$C_{D,f} = \frac{24\Omega}{Re_0} Y_E \tag{21}$$

where $Re_0 = \rho u_{ch} d/\eta_0$ and Y_E is the correction factor for the Ellis fluid.

For a gas bubble ($\Omega = 2/3$) and for $El > 10$, where $El = 2^{1/2}\eta_0 u_{ch}/(\tau_{1/2} d)$ is the Ellis number, Kawase and Moo-Young [16] recommended the relationship

$$Y_E = (3El^2)^{(1-\alpha)/2} \frac{-11 + 28\alpha - 8\alpha^2}{(4-\alpha)(5-2\alpha)} \tag{22}$$

Moreover, the values of the quantity Y_E are physically realistic only in the range of parameter $\alpha < 2.5$, hence with regard to all the above limitations, relationship (22) has only approximate validity. As it is $\tau_{w,p}/\tau_{1/2} = 2^{1/2} Y_E El$, and $\dot{D}_{w,p}\eta_0/\tau_{1/2} = 2^{1/2}El$, relationships (9) and (20) can be modified to give Eq. (21), which can be compared with relationship (22). This comparison for selected values of parameter $\alpha = 1.5$ and $\alpha = 2.4$ is presented in Fig. 2.

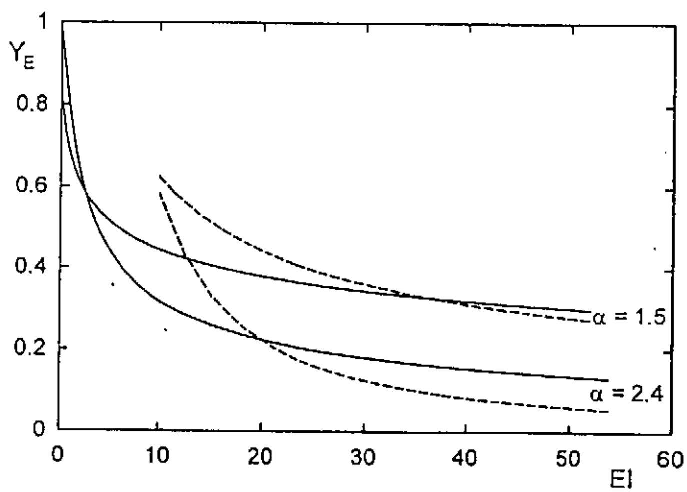

Fig. 2 Comparison of our treatment of creeping motion of gas sphere in Ellis fluid with solution given in literature: ——— our treatment; – – – – Kawase and Moo Young [16]

A qualitatively analogous relationship was suggested by Kawase and Moo-Young [16] also for the three-parameter Carreau flow model (e.g. [17]) containing also zero shear viscosity η_0 as one of its parameters. Because for the above-mentioned flow model Eq. (1) must be solved numerically, the Ellis flow model is — for practical reasons — more suitable if equations of the Rabinowitsch–Mooney type are adopted.

For Newtonian liquid spheres moving slowly in power-law fluid the solution of Eqs (9) and (11) yields

$$Y_{(n)}\Omega = Y_{(n)}\frac{2+3\beta}{3+3\beta} \tag{23}$$

where $Y_{(n)}$ is given by Eq. (14) and

$$\beta \equiv \frac{\mu_f}{\mu_{e,p}} = \frac{\mu_f}{K\left(\frac{2-n}{n}\right)^n\left(\frac{2u_{ch}}{d}\right)^{n-1}} \tag{24}$$

The results of numerical solution for the so-called upper and lower bounds of quantity $Y_{(n)}\Omega$ by Mohan (according to Ref. [17]) are depicted in Fig. 3, where the parameter β_1 is given as

$$\beta_1 \equiv \frac{\mu_f}{K\left(\frac{2u_{ch}}{d}\right)^{n-1}} = \beta\left(\frac{2-n}{n}\right)^n \tag{25}$$

With the use of Eq. (25) it is possible to determine the value of quantity β, and thereby with the use of Eqs (23) and (14) it is also possible to obtain the value of the quantity $Y_{(n)}\Omega$, whose dependence on the parameter n for given values of quantity β_1 is depicted in Fig. 3 too.

In practical calculations it is necessary to determine the value of the quantity Ω depending on the value of the effective viscosity $\mu_{e,p}$ which is defined by Eqs (1) and (4). Because the relationship for consistency variable $\tau_{w,p}$ appearing in Eqs (1) and (4) is given as

$$\tau_{w,p} \equiv \frac{f_t}{\Phi_p} = \frac{f_{t,f}}{\Omega\Phi_p} \tag{26}$$

the method of successive approximations must be used if the velocity of liquid

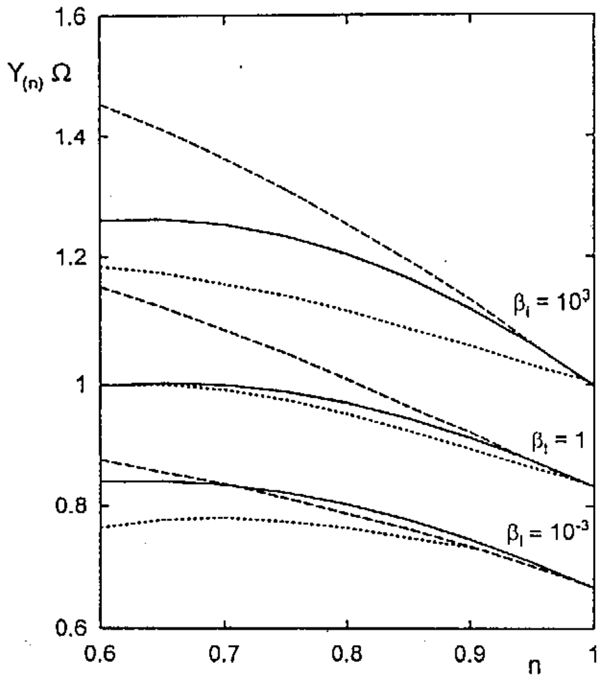

Fig. 3 Comparison of our treatment of creeping motion of liquid sphere in power-law fluid with solution for upper and lower bounds by Mohan (according to Ref. [17]): ——— our treatment; – – – – upper bound; ······ lower bound

spheres is to be determined. With respect to $2/3 \le \Omega \le 1$ we can choose $\Omega_0 = 5/6$ in Eq. (26) for the purposes of determination of the value of quantity $\tau_{w,p,0}$.

Viscoplastic Fluids

For the Robertson–Stiff [18] flow model

$$\dot{D}(\tau) = \left[\frac{\tau}{K'}\right]^{1/n'} - \dot{D}_0 \equiv \left[\frac{\tau}{K'}\right]^{1/n'} - \left[\frac{\tau_0}{K'}\right]^{1/n'} \tag{27}$$

where K', n' and $\dot{D}_0$ are its parameters, the solution of Eq. (1) (in the dimensionless form) yields

$$\dot{D}_{w,p,d} \equiv \frac{2u_{ch}}{d\dot{D}_0} = \frac{n'\xi^{-1/n'}(1 - \xi_p^{1/n'-1/2})}{2 - n'} + 1 - \xi_p^{-1/2} \tag{28}$$

If in the case of a single solid spherical particle the velocity u_{ch} in wall-unbounded system is known, the quantity f_t is determined using Eq. (28) and the momentum balance in the dimensionless form

$$\frac{f_t}{\tau_0} - \frac{1}{\xi_p}(1 + \Psi_p + \xi_p^{1/2}) = 0 \tag{29}$$

where for the cases of both solid and fluid spheres we introduce $f_t = f_{t,f}/\Omega$ (Eq. (9)). The value of the quantity $\tau_{w,p}$ needed for calculation of the effective viscosity $\mu_{e,p}$ and hence also for calculation of the value of the drag correction factor Ω is found from the value of the quantity ξ_p.

If the aim of calculation is to determine the fall or rise velocity of the sphere, the momentum balance is adopted in the form

$$\xi_p = \left[\frac{\varphi_p + \left[\varphi_p^2 + 4\varphi_p(1 + \Psi_p)\right]^{1/2}}{2}\right]^2 \tag{30}$$

where $\varphi_p \equiv \tau_0/f_t = \tau_0\Omega/f_{t,f}$.
Analogously, it is possible to obtain an analytical solution to the Rabinowitsch–Mooney equation (1) as well as to the Casson [19] flow model. For the Bingham flow model (Eq.(27), where $n' = 1$ and $K' = \mu$), the solution to Eq. (28) with $n' = 1$ and that to the momentum balance (Eq. (30)) lead to the relationship

$$f_{t,d} \equiv \frac{f_{t,fd}}{\Omega} = (Bn^{1/2} + 2^{1/2})\left[(2 + \Psi_p)Bn^{1/2} + (1 + \Psi_p)2^{1/2}\right] \tag{31}$$

where $f_{t,d} = f_t d/(\mu u_{ch})$ is the dimensionless total resistance of a solid sphere, $f_{t,f,d}$ is the dimensionless total resistance of both fluid and solid spheres, and $Bn = \tau_0 d/(\mu u_{ch})$ is the Bingham number. For a creeping region of flow ($\Psi_p = 1/2$) and an NF ($Bn = 0$) it is $f_{t,d} = 3$.

The comparisons of Eq. (31) in the form valid for the creeping region of flow ($\Psi_p = 1/2$), with the numerical solution by Yoshioka *et al.* [20] for a solid sphere (variational principle method, where the arithmetic mean of upper and lower bounds of the quantity $f_{t,d}$ was considered), with that by Beris *et al.* [21] (finite element method), and with that by Blackery and Mitsoulis [22] approximated by the following equation with $\Omega = 1$,

$$\frac{f_{t,f,d}}{3\Omega} = 1 + 2.93 Bn^{0.83} \tag{32}$$

are presented in Fig. 1. For fluid spheres moving slowly in VPF, the relationship so far available is Eq. (33) suggested by Bhavaraju *et al.* [6] for a spherical bubble ($\Omega = 2/3$), which is also depicted in Fig. 1.

$$\frac{f_{t,f,d}}{3\Omega} = 1 + 1.61 Bn \tag{33}$$

The quantity Ψ_p is generally calculated with the help of some form of the dependence $C_D = f(Re_p)$ for the fall of a solid spherical particle in NF given in literature (e.g. Ref. [23]), using the relationship

$$\Psi_p = \frac{C_D Re_p}{16} - 1 \tag{34}$$

The value of a fall and a rise velocity of solid ($\Omega = 1$) and gas ($\Omega = 2/3$) spheres in the Robertson–Stiff fluid is then determined using Eqs (28), (7), (30) and (34) by the method of successive approximations. With respect to the fact that $0 < \xi_p < 1$ we can choose $\xi_{p,0} = 1/2$ in Eqs. (28) and (7). In the case of liquid spheres, also the quantity Ω depending on the value of the effective viscosity $\mu_{e,p}$ must be determined by the method of successive approximations. The values of consistency variables needed for calculation of the effective viscosity $\mu_{e,p,0}$ for the value $\xi_{p,0} = 1/2$ chosen are obtained from the definition relationship of the quantity ξ_p ($\tau_{w,p,0} = 2\tau_0$) and from Eq. (28). With the use of the values of quantities $\Psi_{p,0}$ and Ω_0 thus obtained, and with the use of Eq. (30) we can calculate the first approximate value of the quantity $\xi_{p,1}$.

However, the method of calculation suggested can probably be used only up to a certain critical value of the Reynolds number Re_p from which the so-called secondary movement becomes significant. In such case the trajectory of the moving particle is not a straight line but a spiral line. In the region of secondary

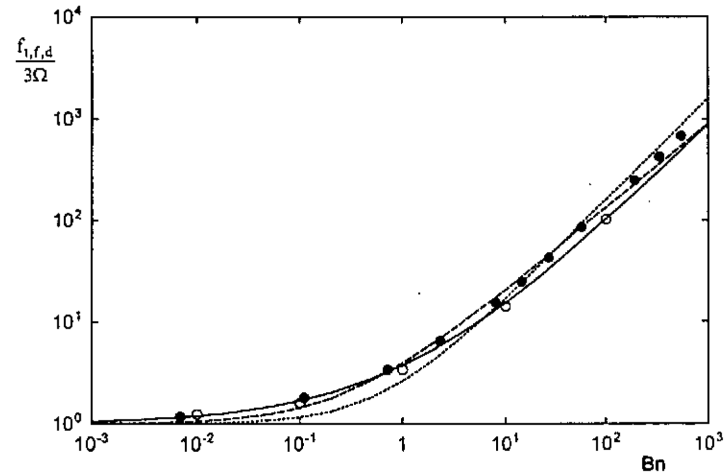

Fig. 4 Comparison of our treatment for creeping motion of single sphere in Bingham fluid with solutions given in the literature: ○ – Yoshioka *et al.* [20]; ● – Beris *et al.* [21]; – – – – Blackery and Mitsoulis [22] (all solid sphere); ······ Bhavaraju *et al.* [6] (gas sphere); ——— our treatment

movement the drag coefficient C_D does not depend only on the Reynolds number Re_p but also on the value of the quantity ρ_p/ρ, and hence the dependence $C_D = f(Re_p)$ valid for a falling particle ($\rho_p/\rho > 1$) cannot be used for a rising particle ($\rho_p/\rho < 1$) in this region (e.g. Ref. [13]) .

Conclusion

An approach has been suggested to the approximate solution of creeping motion of a single fluid and solid sphere in both GNF and VPF. Because the presented equation of the Rabinowitsch–Mooney type together with the corresponding relationships for consistency variables has — in its application — the meaning of an integral form of the rheological equation of state of non-Newtonian fluids, it can be used for any flow models of GNF and VPF. At the same time, the ratio of consistency variables define the effective viscosity of the system which can be substituted for the dynamic viscosity of continuous phase in both the Reynolds number and the relationship suggested by Haddamard–Rybzynski (according to Ref. [1]). This universality and comprehensiveness of the approach suggested also represents its main advantage as compared with the procedures used so far.

Symbols

Bn Bingham number
C_D drag coefficient
d diameter of spherical particle, m
$\dot{D}$ shear rate, s^{-1}
$\dot{D}_0$ parameter of Robertson-Stiff flow model, s^{-1}
$\dot{D}_{w,p}$ kinematic consistency variable for fall of single particle in Eq. (1), s^{-1}
El Ellis number
f_{fr} frictional resistance referred to particle surface area, Pa
f_s shape resistance referred to particle surface area, Pa
f_t total resistance of solid particle referred to its surface area, Pa
$f_{t,f}$ total resistance of fluid particle referred to its surface area, Pa
g gravitational acceleration, m s^{-2}
K parameter of power-law flow model, Pa s^n
K' parameter of Robertson–Stiff flow model, Pa $s^{n'}$
n parameter of power-law flow model
n' parameter of Robertson–Stiff flow model
Re_p Reynolds number
Re_{PL} Reynolds number for power-law fluid used in literature
Re_0 Reynolds number for Ellis fluid used in literature
u_{ch} characteristic velocity of system for flow past a single particle, m s^{-1}
u_t terminal velocity, m s^{-1}
Y correction factor

α parameter of Ellis flow model
β viscosity ratio (dispersed/continuous phase)
η_0 parameter of Ellis flow model
μ dynamic viscosity, Pa s
$\mu_{e,p}$ effective viscosity of non-Newtonian fluid - single particle system, Pa s
ξ_p ratio of yield stress to dynamic consistency variable
ρ density of fluid, kg m^{-3}
ρ_p density of particle, kg m^{-3}
τ shear stress, Pa
τ_0 yield stress, Pa
$\tau_{1/2}$ parameter of Ellis flow model, Pa
$\tau_{w,p}$ dynamic consistency variable for fall and rise of a single sphere in Eq. (8), Pa
φ_p (= τ_0/f_t) dimensionless yield stress
Φ_p resistance factor
Ψ_p resistance number
Ω drag correction factor

Indexes

c continuous phase
d dimensionless
E related to Ellis fluid
f fluid sphere
l related to literature
n related to power-law fluid
p particle

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