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SAMPLING OF FUNCTION GENERATING SHIFT INVARIANT SPACES

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1. Introduction and preliminaries

Packet based networks can be used also for voice transmission [5], [6]. This transmission can je based on regularly shifted functions. Different shape of these functions leads to the different properties of the signal transmission chain (an error rate, a jitter robustness, a packet loss robustness) and a researcher will find the best shape of the signal regarding required property. Anyhow the basic property must be kept in any case. These regularly shifted functions have to create the basis.

In article [1] we gave the necessary and sufficient condition for function generating basis. The advantage was, that the condition concerned not only the orthogonal basis. The paper gives a new conditions for a function generating a shift invariant space. This condition is more usable in applications, because only sampling values of basis function are studying.

The decomposition

$$f(t) = \sum_{k \in Z} f(k\Delta)\,\mathrm{sinc}(\Omega(t - k\Delta)), \quad t \in R, \quad \Delta \in \frac{\pi}{\Delta}$$

is known as a Shanon sampling theorem. Complex view on this theorem can be found in [3]. This theorem is one of the models of basis generating by shifting of one function.

The generating function in this case is a function sinc:

$$\text{sinc} = \frac{\sin(t)}{t}, \quad t \in R - \{0\}$$
$$= 0, \qquad t = 0$$

The set

$$\{\Phi_k(t) = \text{sinc}(\Omega(t - k\Delta)), \quad t \in R, \quad k \in Z\}$$

creates the orthogonal basis of the Hilbert space $S \subset L_2(-\infty, \infty)$ of all the Ω -band limited functions, i.e. the functions, which Fourier transform is equal zero out of the interval $(-\Omega, \Omega)$.

Similar problems are solved in wavelet theory [2].

The important notion of the theory of basis in infinity dimensional vector spaces is Ω -independent of system of the functions, see in [4].

A system of the functions $\{\Phi_k\} \subset L_2(-\infty, \infty)$ *is called* ω *-independent if and only if*

$$\sum_{k=-\infty}^{\infty} c_k \Phi_k = \theta \qquad \Rightarrow \qquad c_k = 0, \quad \forall k \in Z$$

We can see the necessary and sufficient condition for function $\Phi(t)$, which is generating the ω - independent system of the functions in the following theorem.

Theorem 1.1 *Let a function* $\Phi(t) \in L_2(-\infty, \infty)$ *be the continuous function and* Δ *be a positive constant. Then the function* $\Phi(t)$ *generates the* ω *-independent system* $\{\Phi_k\}$ *if and only if*

$$\sum_{n \in Z} | \hat{\Phi}(\omega + \frac{2\pi n}{\Delta}) | \neq 0, \quad \omega \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle, \tag{1}$$

where

$$\{\Phi_k(t) = \Phi(t - k\Delta) \subset L_2(-\infty, \infty), \quad t \in R, \quad k \in Z\}$$

and $\hat{\Phi}(\omega)$ *is the Fourier transform of the function* $\Phi(t)$:

$$\hat{\Phi}(\omega) = \int_{-\infty}^{\infty} \Phi(t) e^{-j\omega t} dt, \qquad \omega \in R$$

The proof of this theorem can be found in [1]. Now, we give some notice about using theorem 1.1. The case of $\Delta = 0$ is not very interesting because any $\Phi(t)$

$$\Phi(t) \in L_2(-\infty, \infty), \quad \Phi(t) \neq 0, \quad \forall \omega \in R,$$

creates one dimensional space of functions $f(t) \in L_2(-\infty, \infty)$: $f(t) = c\Phi(t), \quad c, t \in R$.

Not all of the functions $\Phi(t)$ satisfying the conditions of the theorem1.1 are suitable as a basis generator in the real applications.

The coordinates c_k in the scanning $f(t)=\sum_{k=-\infty}^{\infty} c_k \Phi_k(t)$ are calculated using

$$\left\langle f(t),\Phi_k(t)\right\rangle = \left\langle \sum_{n=-\infty}^{\infty} c_n \Phi_n(t),\Phi_k(t)\right\rangle, \qquad \forall k \in Z$$

and after simplify, as a solution of a system of equations

$$\left\langle f(t),\Phi_k(t)\right\rangle = \sum_{n=-\infty}^{\infty} c_n \left\langle \Phi_n(t),\Phi_k(t)\right\rangle, \qquad \forall k \in Z \tag{2}$$

where the variables are c_n and coefficients of the system are

$$\left\langle f(t),\Phi_k(t)\right\rangle = \int_{-\infty}^{\infty} f(t)\Phi(t-k\Delta)dt\ ,\ \left\langle \Phi_n(t),\Phi_k(t)\right\rangle = \int_{-\infty}^{\infty} \Phi(t-n\Delta)\Phi(t-k\Delta)dt \quad \forall k,n \in Z$$

The system (2) generally contains of the unlimited number of equations and unlimited number of variables and it is not easy to solve it.

Next problem is, that the integrals used in calculating of coefficients $\left\langle f(t),\Phi_k(t)\right\rangle$ and $\left\langle \Phi_n(t),\Phi_k(t)\right\rangle$ are not necessary finite. If we obtain function $f(t)$ as a signal from a real source, then we are able to calculate c_k only if there exists such constant $T \in R$ (or in spectral domain such constant $\Omega \in R^+$, that

$f(t)=0,\quad \forall |t|>T,\quad t\in R$, (or spectral condition $\hat{f}(\omega)=0,\quad \forall|\omega|>\Omega,\quad \omega \in R$) i.e. constant T (or Ω) denotes the end of the signal generating (or the maximum value of frequency). Those conditions are usually satisfied for the real time communication that we are mainly interested in.

The support boundary of the signal in the spectral domain is used in the Shanon basis. In our interest will be support boundary in the time domain:

$$\exists T \in R^+;\quad \Phi(t)=0,\quad \forall |t|>T, t\in R \tag{3}$$

Theorem 1.2 *Let the function* $\Phi(t)$ *satisfies the condition* (3), *then every equation of the system* (2) *contains only finite number of non zero coefficients*.

Proof:

The coefficients of the system

$$\left\langle f(t),\Phi_k(t)\right\rangle = \left\langle \sum_{n=-\infty}^{\infty} c_n \Phi_n(t),\Phi_k(t)\right\rangle, \qquad \forall k \in Z$$

are equal

$$\left\langle \Phi_n(t),\Phi_k(t)\right\rangle = \int_{-\infty}^{\infty} \Phi(t-n\Delta)\Phi(t-k\Delta)dt \quad \forall k,n \in Z$$

If $|n-k|>2T$ then a product of $\Phi(t-n\Delta)\Phi(t-k\Delta)$ is equal to zero, so

$$\langle \Phi_n(t), \Phi_k(t) \rangle = 0 \quad \forall k,n \in Z; \quad | n-k | > 2T$$

Theorem 1.3 *Let the function* $\Phi(t) \in L_2(-\infty,\infty)$ *has Fourier transform* $\hat{\Phi}(\omega) \in L_2(-\infty,\infty)$. *Let the function* $\Phi(t)$ *satisfies the condition* (3). *Then the next two conditions are equivalent*

$$\sum_{n \in Z} | \hat{\Phi}(\omega + \frac{2\pi n}{\Delta}) | \neq 0 \qquad \forall \omega \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle \tag{1}$$

$$\sum_{n \in Z} | \hat{\Phi}(\frac{\pi k}{T} + \frac{2\pi n}{\Delta}) | \neq 0 \qquad \forall \frac{\pi k}{T} \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle \tag{4}$$

where ω ,*T*>0 *are real numbers and K is suitable integer number*.

Proof:

Without loss of generality, we can suppose that $T = m\Delta$, where m is suitable integer number. It is clearly that if $\sum_{n \in Z} | \hat{\Phi}(\omega + \frac{2\pi n}{\Delta}) | \neq 0$ satisfies for $\forall \omega \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle$, it satisfies for some $\omega^* = \frac{\pi k^*}{T} \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle$, because ω^* is one of the ω.

So (1) $\Rightarrow$ (4).

Now, we are going to prove implication (4) $\Rightarrow$ (1). The contrapositive version of this implication is: Not (1) $\Rightarrow$ Not (4)

$$\exists \omega_0 \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle \ \sum_{n \in Z} | \hat{\Phi}(\omega_0 + \frac{2\pi n}{\Delta}) | = 0 \ \Rightarrow$$

$$\exists \frac{\pi k_0}{T} \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle \ \sum_{n \in Z} | \hat{\Phi}(\frac{\pi k_0}{T} + \frac{2\pi n}{\Delta}) | = 0$$

Assume, there exists such $\omega_0 \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle$, for which $\sum_{n \in Z} | \hat{\Phi}(\omega_0 + \frac{2\pi n}{\Delta}) | = 0$

then $0 = \hat{\Phi}(\omega_n) = \hat{\Phi}(\omega_0 + \frac{2\pi n}{\Delta})$.

According the condition $\Phi(t) \in L_2(-\infty,\infty)$, $\Phi(t)$ has Fourier transform $\hat{\Phi}(\omega)$ and by the symetry property of Fourier transform

$$\hat{\Phi}(t) \ \leftrightarrow \ 2\pi \Phi(-\omega)$$

The function $\Phi(t)$ holds (3), then the function $2\pi\Phi(-\omega)$ satisfies condition (3) too.

It means that, the function $\hat{\Phi}(\omega) \in L_2(-\infty,\infty)$ \$ is *T*-band limited and by the Shanon sampling theorem:

$$0 = \hat{\Phi}(\omega_n) = \sum_{k \in Z} \hat{\Phi}(\frac{\pi k}{T}) \operatorname{sinc} T(\omega_n - \frac{\pi k}{T}) \qquad \forall n \in Z \tag{5}$$

then $$0 = \hat{\Phi}(\omega_n) = \hat{\Phi}(\omega_0) \qquad \forall n \in Z$$

$$\sum_{k \in Z} \hat{\Phi}(\frac{\pi k}{T}) \operatorname{sinc} T(\omega_0 - \frac{\pi k}{T}) = \sum_{k \in Z} \hat{\Phi}(\frac{\pi k}{T}) \operatorname{sinc} T(\omega_n - \frac{\pi k}{T}) =$$

$$= \sum_{k \in Z} \hat{\Phi}(\frac{\pi k}{T}) \operatorname{sinc} T(\omega_0 + \frac{2\pi n}{\Delta} - \frac{\pi k}{T}) =$$

$$= \sum_{k \in Z} \hat{\Phi}(\frac{\pi k}{T}) \operatorname{sinc}(T\omega_0 + 2\pi nm - k\pi) = \sum_{k \in Z} \hat{\Phi}(\frac{\pi k}{T}) \operatorname{sinc}(T\omega_0 + (2nm - k)\pi) =$$

$$= \sum_{l \in Z} \hat{\Phi}(\frac{\pi(2nm + l)}{T}) \operatorname{sinc}(T\omega_0 - l\pi) = \sum_{l \in Z} \hat{\Phi}(\frac{\pi l}{T} + \frac{2\pi n}{\Delta}) \operatorname{sinc} T(\omega_0 - \frac{l\pi}{T})$$

For all $n \in Z$ holds

$$\sum_{k \in Z} \hat{\Phi}(\frac{\pi k}{T}) \operatorname{sinc} T(\omega_0 - \frac{\pi k}{T}) = \sum_{l \in Z} \hat{\Phi}(\frac{\pi l}{T} + \frac{2\pi n}{\Delta}) \operatorname{sinc} T(\omega_0 - \frac{l\pi}{T})$$

From this equation and the properties of Shanon sampling can be seen, that the function $\hat{\Phi}(t)$ is periodical, with the period $\frac{2\pi n}{\Delta}$. But only periodical function $\Phi(t) \in L_2(-\infty,\infty)$ is $\Phi(t) = \theta$. Clearly

$$\exists \frac{\pi k_0}{T} \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle \text{ such, that } \quad \sum_{n \in Z} | \hat{\Phi}(\frac{\pi k_0}{T} + \frac{2\pi n}{\Delta}) | = 0$$

This completes the proof.

Example 1.1 *Let* $\Phi(t) \in L_2(-\infty,\infty)$ *be the continuous function for which* $\Phi(t) > 0$ *for* $0 < |t| < \frac{\Delta}{2}$ *and* $\Phi(t) = 0$ *for* $|t| > \frac{\Delta}{2}$, *then the* $\Phi(t)$ *generates the* ω*-independent system*

$$\{\Phi_k(t) = \Phi(t - k\Delta), \quad t \in R, \quad k \in Z\}$$

Condition (4) is satisfied because

$$\sum_{n \in Z} | \hat{\Phi}(\frac{\pi k}{T} + \frac{2\pi n}{\Delta}) | \neq 0 \qquad \forall \frac{\pi k}{T} \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle$$

$$\frac{\pi k}{T}=\frac{\pi k}{\frac{\Delta}{2}}=\frac{2\pi k}{\Delta}\in\left\langle -\frac{\pi}{\Delta},\frac{\pi}{\Delta}\right\rangle \text{ only for } k=0$$

and condition $\sum_{n\in Z}|\hat{\Phi}(\frac{2\pi n}{\Delta})|\neq 0$ is, under assumptions mentioned above, always true.

The scanning of the signal $f(t)$ is

$$f(t)=\sum_{k=-\infty}^{\infty}c_k\Phi_k(t) \text{ where } \Phi_k(t)=\Phi(t-k\Delta),\ t\in R$$

and the system (2) of equations for calculating the coefficients c_k has form

$$\left\langle f(t),\Phi_k(t)\right\rangle=\sum_{n=-\infty}^{\infty}c_n\left\langle \Phi_n(t),\Phi_k(t)\right\rangle \qquad \forall k\in Z$$

The inner product of the functions $\Phi_n(t)$, $\Phi_k(t)$ is equal to zero for all $n\neq k$, hence the system (2) contains only one non zero coefficient in each equation:

$$\left\langle f(t),\Phi_k(t)\right\rangle=c_k\left\langle \Phi_k(t),\Phi_k(t)\right\rangle \qquad \forall k\in Z$$

If we denote as A the integral $\int_{-\infty}^{\infty}\Phi(t)\Phi(t)dt$, we get a formula for account coefficients

$$c_k=\frac{\left\langle f(t),\Phi_k(t)\right\rangle}{\left\langle \Phi_k(t),\Phi_k(t)\right\rangle}=\frac{\left\langle f(t),\Phi_k(t)\right\rangle}{A} \quad .$$

It is the case of function $\Phi(t)$, which is generated the orthogonal basis.

Example 1.2 *Let* $\Phi(t)\in L_2(-\infty,\infty)$ *be the continuous function for which* $\Phi(t)>0$ *for* $0<|t|<\Delta$ *and* $\Phi(t)=0$ *for* $|t|>\Delta>0$*, then the* $\Phi(t)$ *generates the* ω*-independent system* $\{\Phi_k(t)=\Phi(t-k\Delta),\quad t\in R,\quad k\in Z\}$ *if and only if*

$$\sum_{n=-\infty}^{\infty}|\hat{\Phi}(\frac{2\pi n}{\Delta})|\neq 0 \ \textit{and} \ \sum_{n=-\infty}^{\infty}|\hat{\Phi}(\frac{\pi(2n+1)}{\Delta})|\neq 0$$

The scanning of the signal $f(t)$ is

$$f(t)=\sum_{k=-\infty}^{\infty}c_k\Phi_k(t) \text{ where } \Phi_k(t)=\Phi(t-k\Delta),\ t\in R$$

The inner product of the functions $\Phi_n(t)$ and $\Phi_k(t)$ equals:

$$\left\langle \Phi_n(t),\Phi_k(t)\right\rangle=0 \quad \text{for} \quad |n-k|>1$$

hence the system (2) contains only three non zero coefficients in each equation:

$$\langle f(t),\Phi_k(t)\rangle = c_{k-1}\langle \Phi_{k-1}(t),\Phi_k(t)\rangle + c_k\langle \Phi_k(t),\Phi_k(t)\rangle + c_{k+1}\langle \Phi_{k+1}(t),\Phi_k(t)\rangle \quad \forall k \in Z$$

Now we denote $A_k = \langle f(t),\Phi_k(t)\rangle$ and $B_{kl} = \langle \Phi_k(t),\Phi_l(t)\rangle$ for $\forall k,l \in Z$.

Clearly $\quad B_{kl} = 0$ for $|k-l|>1$

$$B_{kl} = \int_{-\infty}^{\infty}\Phi(t)\Phi(t-\Delta)dt \equiv B_1 \text{ for } |k-l|=1$$

$$B_{kl} = \int_{-\infty}^{\infty}\Phi(t)\Phi(t)dt \equiv B_0 \text{ for } k=l$$

We get a recursion formula to calculate coefficients c_k :

$$\begin{array}{llll} B_1c_1 = A_0 - & B_1c_{-1} - & B_0c_0 & \\ B_1c_2 = A_1 - & B_1c_0 - & B_0c_1 & \\ \vdots \qquad \vdots & & \vdots & \\ B_1c_k = A_{k-1} - & B_1c_{k-2} - & B_0c_{k-1} & k \in Z \end{array}$$

In this case the function $\Phi(t)$ does not generate the orthogonal basis.

2. Conditions for the basis orthogonality

The proof of the next condition can be found in [1]:

Theorem 2.1 *Let* $\Delta > 0$ *. The function* $\Phi(t) \in L_2(-\infty,\infty)$ *generates the orthonormal basis.*

$\{\Phi_k(t) = \Phi(t-k\Delta), \quad t \in R, \quad k \in Z\}$ *if and only if*

$$\sum_{n\in Z} |\hat{\Phi}(\omega + \frac{2\pi n}{\Delta})|^2 = \Delta, \quad \omega \in \left\langle -\frac{\pi}{\Delta},\frac{\pi}{\Delta}\right\rangle$$

Theorem 2.1 *Let* $\Delta > 0$ *and let the function* $\Phi(t) \in L_2(-\infty,\infty)$ *has the Fourier transform* $\hat{\Phi}(\omega) \in L_2(-\infty,\infty)$ *and function* $\Phi(t)$ *satisfies the condition*

$$\exists T \in R^+; \quad \Phi(t) = 0, \quad \forall |t| > T, \quad t \in R, \quad T = m\Delta$$

The system of functions $\{\Phi_k(t) = \Phi(t-k\Delta), \quad t \in R, \quad k \in Z\}$ *generated by the function* $\Phi(t)$ *is the orthonormal basis of some subspace* $S \subset L_2(-\infty,\infty)$ *if and only if*

$$\sum_{n\in Z} |\hat{\Phi}(\frac{\pi k}{T}+\frac{2\pi n}{\Delta})|^2=\Delta \qquad (7)$$

$$\text{for } k\in Z \text{ that } \frac{\pi k}{T}\in\left\langle -\frac{\pi}{\Delta},\frac{\pi}{\Delta}\right\rangle.$$

Proof:

Sufficient condition:

the system of functions $\{\Phi_k(t) \quad k\in Z\}$ generates the orthonormal basis, then according theorem 2.1. for every real $\omega\in\left\langle -\frac{\pi}{\Delta},\frac{\pi}{\Delta}\right\rangle$ is

$$\sum_{n\in Z} |\hat{\Phi}(\omega+\frac{2\pi n}{\Delta})|^2=\Delta$$

then for $\frac{\pi k}{T}\in\left\langle -\frac{\pi}{\Delta},\frac{\pi}{\Delta}\right\rangle$ is

$$\sum_{n\in Z} |\hat{\Phi}(\frac{\pi k}{T}+\frac{2\pi n}{\Delta})|^2=\Delta$$

Necessary condition:

Let for $k\in Z$ such, that $\frac{\pi k}{T}\in\left\langle -\frac{\pi}{\Delta},\frac{\pi}{\Delta}\right\rangle$ holds

$$\sum_{n\in Z} |\hat{\Phi}(\frac{\pi k}{T}+\frac{2\pi n}{\Delta})|^2=\Delta$$

where

$$\frac{\pi k}{T}=\frac{\pi k}{m\Delta}\in\left\langle -\frac{\pi}{\Delta},\frac{\pi}{\Delta}\right\rangle \Rightarrow k\in(-m,m);\ k\in Z$$

The function $S(\omega)=\sum_{n\in Z} |\hat{\Phi}(\omega+\frac{2\pi n}{\Delta})|^2$ is defined for $\omega\in\left\langle -\frac{\pi}{\Delta},\frac{\pi}{\Delta}\right\rangle$ and for $\omega=\frac{\pi k}{T}$ is equal to $S(\frac{\pi k}{T})=\Delta$.

We are going to show, that only constant function $S(\omega)=\Delta$ satisfies those conditions:

The Fourier expansion of the function $S(\omega)$, $\omega\in\left\langle -\frac{\pi}{\Delta},\frac{\pi}{\Delta}\right\rangle$ is

$$S(\omega)=\sum_{l\in Z} c_l e^{j\omega l\Delta} \quad \text{where } c_l=\frac{\Delta}{2\pi}\int_{-\frac{\pi}{\Delta}}^{\frac{\pi}{\Delta}} S(\omega)e^{-j\omega l\Delta}d\omega$$

then

$$c_l=\frac{\Delta}{2\pi}\int_{-\frac{\pi}{\Delta}}^{\frac{\pi}{\Delta}}\sum_{n\in Z}|\hat{\Phi}(\omega+\frac{2\pi n}{\Delta})|^2\; e^{-j\omega l\Delta}d\omega=\frac{\Delta}{2\pi}\sum_{n\in Z}\int_{-\frac{\pi}{\Delta}}^{\frac{\pi}{\Delta}}|\hat{\Phi}(\omega+\frac{2\pi n}{\Delta})|^2\; e^{-j\omega l\Delta}d\omega=$$

$$=\frac{\Delta}{2\pi}\int_{-\infty}^{\infty}|\hat{\Phi}(\omega)|^2\; e^{-j\omega l\Delta}d\omega=\Delta\int_{-\infty}^{\infty}\Phi(t)\Phi(t-l\Delta)e^{-j\omega l\Delta}dt=\left\langle\Phi_0,\Phi_l\right\rangle$$

the integral $\int_{-\infty}^{\infty}\Phi(t)\Phi(t-l\Delta)e^{-j\omega l\Delta}dt$ equals to zero for $l\Delta>2T$, or for $l>2m$.

It means only $2m-1$ coefficients c_l can be nonzero.

The function $S(\omega)$ can be rewritten as $S(\omega)=\sum_{l=-(m-1)}^{m-1} c_l e^{j\omega l\Delta}$ where $c_l=\left\langle\Phi_0,\Phi_l\right\rangle$

and according $S(\frac{\pi k}{T})=\Delta$

$$\Delta=S(\frac{\pi k}{T})=\sum_{l=-(m-1)}^{m-1} c_l e^{j\frac{\pi k}{T}l\Delta}=\sum_{l=-(m-1)}^{m-1} c_l e^{j\frac{kl}{m}\pi} \quad \text{where } k\in Z,\quad -m<k<m$$

we get the system of the linear equations

$$\begin{aligned}
\Delta &= \sum_{l=-(m-1)}^{m-1} c_l e^{j\frac{-(m-1)l}{m}\pi}\\
\Delta &= \sum_{l=-(m-1)}^{m-1} c_l e^{j\frac{-(m-2)l}{m}\pi}\\
&\vdots\\
\Delta &= \sum_{l=-(m-1)}^{m-1} c_l e^{j\frac{(m-1)l}{m}\pi}
\end{aligned}\tag{8}$$

The system (8) has only one solution $\mathbf{c}=(c_{-(m-1)},c_{-(m-2)},\dots,c_{(m-1)})$, because the vectors of the coefficients of the rows of the system (8)

$$\mathbf{v_k}=(e^{j\frac{-(m-1)k}{m}\pi},e^{j\frac{-(m-2)k}{m}\pi},\dots,e^{j\frac{(m-1)k}{m}\pi})$$

are linearly independent and then the determinant of this system is not equal to zero.

So there exists only one function $S(\omega)$, such that $S(\frac{\pi k}{T})=\Delta$.

$$S(\omega) = \Delta \quad for \qquad \omega \in \left\langle -\frac{\pi}{\Delta}, \frac{\pi}{\Delta} \right\rangle$$

and according theorem 2.1 the function $\Phi(t)$ creates the orthonormal system $\{\Phi_k(t)\}$.

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Resumé

VZORKOVANIE FUNKCIE GENERUJÚCEJ BÁZU

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Digitálne telefónne siete sú založené na prenose vzoriek reči. Výkonnosť dnešných procesorov umožňuje výpočet všeobecných koeficientov rozkladu reči do priestorov generovaných posunutými funkciami.

V práci [1] bola uvedená podmienka pre funkciu, ktorej posúvaním vznikne ω-nezávislá množina. V tomto článku je podmienka modifikovaná pre použitie v aplikáciách týkajúcich sa prenosu hlasu internetom (VoIP), kedy sa nepoužíva celý priebeh signálov, ale len hodnoty vo vzorkovacích intervaloch.

Summary

SAMPLING OF FUNCTION GENERATING SHIFT INVARIANT SPACES

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Digital telephone networks are based on speech samples transmission. Today's processor performance allows calculate coefficient in more general sense based on speech decomposition to the shift invariant spaces.

In the article [1] are given conditions for the function that generated basis. The new conditions are presented in this article. The conditions are very useful in the applications concerning VoIP, because only sampling values of basis function are studying.

Zusammenfassung

DIE ABTASTUNG DER BASEGENERIERTE FUNKTION

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Die digitalen Telephonnetze sind an der Abtastübertragung der Sprache gebildet. Die Leistungsfähigkeit die gleichzeitigen Prozessoren erlaubt Berechnung der allgemeinen Kenzahlen für Sprachzerlegung an die Raume, die sind durch Funktionverschiebung generiert.

In Arbeit [1] wurde eine Vorbedingung für Funktion, deren Verschiebung eine ω-unabhängige Menge erwähnt. In diesem Artikel ist die Vorbedingung für Anwendung in VoIP-bezogene Applikationen modifiziert, in deren nicht ganzen Signalablauf, sondern nur die Werte in einzelnen Abtasten benutzen werden.