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OPTIMAL LOCATION OF DISTRIBUTION CENTERS UNDER UNCERTAIN COSTS

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Annotation

It is fairly questionable to estimate future costs in the problems of strategic decision. The uncertainty may cause that a resulting solution could be very inefficient considering current costs. In this report we try to find an approach, which enables to determine a unique solution of a location problem under uncertain costs so that the solution be resistant to future changes. We deal with a sensitivity analysis and with a connection of an exact mathematical programming method and the theory of fuzzy sets.

1. Introduction

In managerial experience we can find a problem of service centre optimal location. Location of those objects like manufactures, distributive and shopping centers, supply depots markedly affects the costs of material flows in creative logistic networks. The location of centers is so much complicated because there is not only one logistic chain but a whole distributive network.

Determination about a location or non-location of a service centre in some areas will affect the systems effectiveness for next several years. For finding the optimal solution it is possible to apply an exact method, but only for the known costs. When we solve location problems, for the most of them we have no real future costs, only their gross estimates. So it is necessary to deal with the approach of a location problem solving under uncertain costs.

This paper deals with a possible method of finding the optimal location of service centers under uncertain costs represented by fuzzy numbers.

2. Location problem formulation

A service centre can be set up only in some places from the finite set of possible locations, which requires standby costs. In the system are also costs of satisfying customer demands from some of located canters, which depend on quantity of requirements. The goal is to minimize complete costs of the system. So we have a difficult combinatorial problem of determination of a located service canters number.

There is securing freight traffic from one or more primary centers to customers in the distribution system. This freight traffic could be linear (without transshipments) or combined with transshipments in some centers called terminals, which are often warehouses or buffer stocks. The structure of distribution system is figured out by a set of primary centers, customers, terminals and flows of goods among them.

The location problem is a problem of optimal location of service canters on the given part of the transportation network.

The incapacitated location problem is conceived as follows:

The transportation network is given with customers in the nodes $j \in J$ and localities $i \in I$, in which it is possible to locate serve canters. Let's also assume that also one centre located in the node from the set I is able to serve all customers (see ***Fig.1***). The task is to minimize complete costs, which include standby costs f_i paid for each location of the service centre in i and variable costs c_{ij} of demand satisfaction b_{ij} of a customer j from the terminal i. The variable costs for satisfying demand b_j of customer $j \in J$ $c_{ij} = \left(e_1 d_{Si} + e_0 d_{ij} + g_i\right) b_j$ consist of charges e_1 for import from the primary centre S to the terminal i, costs g_i for transshipment in the transshipment i and charges e_0 for freight traffic from i to the customer j. The haul between the primary centre S and the terminal i is d_{Sj} and between the terminal i and the customer j is d_{ij}. The condition is that all the customers have to be served, or more precisely have to be assigned to some of the located terminals.

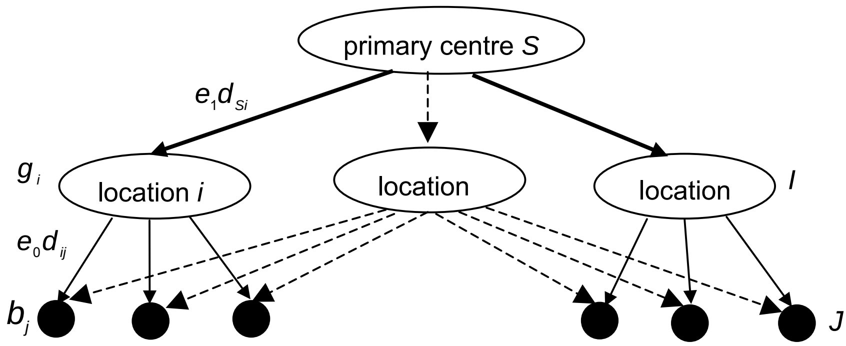

***Fig. 1** Two designs of distribution system with transshipments*

Having introduced 0-1 variable $y_i \in \{0,1\}$ for each $i \in \mathbf{I}$, which models the decision if the terminal is located at *i* or not, and variable $z_{ij} \in \{0,1\}$ for each pair *i, j*, $i \in I$, $j \in J$, which assigns the customer *j* to the terminal location *i*, we can set the following model of the complete cost minimization.

Minimize $f(y,z) = \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij}$ where $c_{ij} = (e_1 d_{Si} + e_0 d_{ij} + g_i) b_j$ (1)

subject to $\sum_{i \in I} z_{ij} = 1$ for $j \in J$ (2)

$$z_{ij} \le y_i \text{ for } i \in I ,\ j \in J \quad (3)$$

$$z_{ij} \ge 0 \text{ for } i \in I ,\ j \in J \quad (4)$$

$$y_i \in \{0, 1\} \text{ for } i \in I \quad (5)$$

where:

$j \in J$ ....network of customers

$i \in I$ ....localities, in which it is possible to locate serve canters

f_i ....standby costs paid for each location of the service centre in *i*

y_i ....0-1 variable which models the decision if the terminal is located in *i* or not

b_j ....demand of a customer *j*

z_{ij} ....0-1 variable which assigns the customer *j* to the terminal location *i*

e_1 ....charges for import from the primary centre S to the terminal *i*

g_i ....costs for transshipment in the terminal *i*

e_0 ....charges for freight traffic from *i* to the customer *j*

d_{Sj} .... distance between the primary centre S and the terminal i

d_{ij}. .... distance between the terminal i and the customer j

In the model above, the objective function (1) represents the complete costs of distribution system. Constraints (2) ensure that each customer demand has to be satisfied from exactly one terminal location, constraints (3) force placement of a terminal at location i whenever a customer is assigned to the terminal location i, constraints (4) ensure the location of terminal in every locality from which demands of some customers are satisfied.

3. Analysis of the existing approaches

In strategic decision problems it is difficult to estimate future values of standby or/and variable costs. In this case, the estimation of future costs is inaccurate. Considering confidential variables, which model determination about (un)location of terminals, the resulting solution can be economically inefficient in the view of the future costs. For example, the growth of f_i or e_1 creates a change of system structure of locations number and change of customer's assignment (see ***Fig.1***). As a consequence, the estimation of expected costs by one numeric value is risky. Uncertain costs can be in that case described by an expectant interval of change of coefficient f_i or c_{ij}, (but the uncertainty is too big) or by fuzzy number, which gives us more information about charges.

There are two approaches how to overcome the uncertainty.

First of them is a classical sensitivity analysis [2], which tells us how the optimal solution changes when some of the parameters has other value than the one which was calculated.

If the uncertain parameter $\boldsymbol{f}_i$ changes in the interval $\left\langle \boldsymbol{f}_i^1, \boldsymbol{f}_i^3 \right\rangle$, by dividing this interval into m parts we will have m+1 location problems with the known costs (6). But the result of sensitivity analysis is not a unique solution.

Fig. 2 *Interval of standby costs*

$$\text{Minimize } f(y,z) = \sum_{i \in I} \boldsymbol{f}_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij} \quad \boldsymbol{f}_i \in \left\langle \boldsymbol{f}_i^1, \boldsymbol{f}_i^3 \right\rangle \tag{6}$$

Another approach uses the theory of fuzzy sets, where the uncertain value q is described by a possible interval and membership function μ_q (see ***Fig.3***) - it is a power of applicability of given element to q. This membership function has a triangular form.

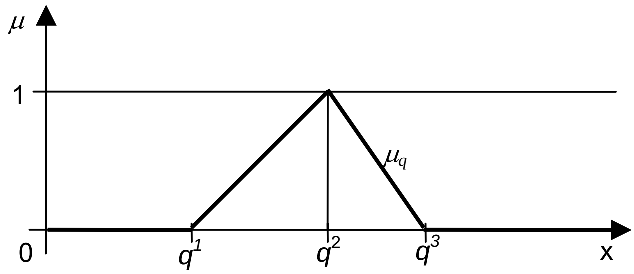

Fig. 3 *Membership function* μ_q

According to fuzzy arithmetic rules, fuzzy numbers can be mutually added, subtracted and multiplied and divided by a real number without loss of the triangular form. When the coefficients q_j of an objective function $F = q_1 x_1 + q_2 x_2 + \ldots + q_n x_n$ of a linear programming problem are triangular fuzzy numbers, then the value of the objective function for a given set of variable values $\boldsymbol{x} = \langle x_1, x_2, \ldots, x_n \rangle$ is also a triangular fuzzy number: $\boldsymbol{x}$

$$\boldsymbol{F}(\boldsymbol{x}) = \langle F^1(\boldsymbol{x}), F^2(\boldsymbol{x}), F^3(\boldsymbol{x}) \rangle = \left\langle \sum_{j=1}^{n} q_j^1 x_j, \sum_{j=1}^{n} q_j^2 x_j, \sum_{j=1}^{n} q_j^3 x_j \right\rangle. \quad (7)$$

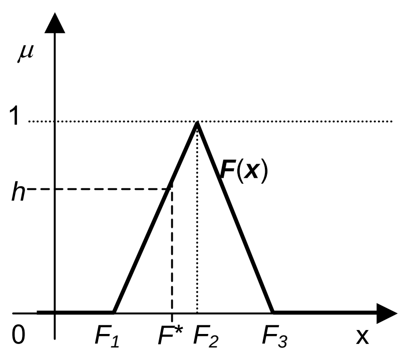

Fig. 4 *Membership function of fuzzy number* ***F***

The existing approach [4], which uses the theory of fuzzy sets looks for a solution for thr given level of satisfaction h which is given by an expert (see ***Fig. 4***). So we solve the original task, but with a changed objective function describing uncertain costs:

$$\text{Minimize} \quad F^*(\boldsymbol{x}) = F^1(\boldsymbol{x}) + h\left(F^2(\boldsymbol{x}) - F^1(\boldsymbol{x})\right) \quad (8)$$

The result of this method is a concrete determination, but credibility of the associated result depends on an expert's ability and his experience in determining a suitable level of satisfaction.

4. Concept of location problem solving

One method for finding a concrete determination about the service centers location, which is not dependent on an expert's ability is the fuzzy algorithm [3]. This approach is based on introducing the fuzzy set $\boldsymbol{F_s}$, which expresses an assertion that ″value of F is small" with the membership function shown in ***Fig. 5***, where $F^{\min}$ and $F^{\max}$ denote respectively minimal values of $F^1(\boldsymbol{x})$ and $F^2(\boldsymbol{x})$ over a set of feasible solutions of the problem.

In this approach we searched a feasible solution $\boldsymbol{x}^*$, for which the membership function of the fuzzy set "$\boldsymbol{F}(\boldsymbol{x})$ and $\boldsymbol{F_s}$" obtains the maximal value h (see ***Fig. 5***).

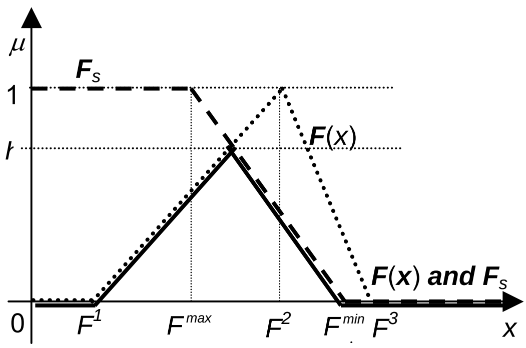

Fig. 5 *Membership function of fuzzy sets* $\boldsymbol{F}(\boldsymbol{x})$, $\boldsymbol{F_s}$ *and their intersection* $\boldsymbol{F}(\boldsymbol{x})$ ***and*** $\boldsymbol{F_s}$

The maximal value h of the membership function of the fuzzy set $\boldsymbol{F}(\boldsymbol{x})$ ***and*** $\boldsymbol{F_s}$ for the given $\boldsymbol{x}$ has to satisfy the following equality in the cases when $F^1(\boldsymbol{x}) \le F^{\max}$ holds.

$$F^1(\boldsymbol{x}) + \left(F^2(\boldsymbol{x}) - F^1(\boldsymbol{x})\right)h = F^{\max} - \left(F^{\max} - F^{\min}\right)h \tag{9}$$

In other cases h can be set to zero. For the former case we get

$$h(\mathbf{x}) = \frac{F^{\max} - F^1(\boldsymbol{x})}{F^2(\boldsymbol{x}) - F^1(\boldsymbol{x}) + F^{\max} - F^{\min}}\,. \tag{10}$$

and we seek for $\boldsymbol{x}^*$ maximizing $h(\boldsymbol{x})$, which is a non-linear programming problem.

The following numerical process [3] obtains an approximate solution of the problem.

1° Set $\underline{h}$ to an initial positive value near zero.

2° Minimize the following objective function $F^1(\boldsymbol{x}) + \left(F^2(\boldsymbol{x}) - F^1(\boldsymbol{x})\right)\underline{h}$ over the set of feasible $\boldsymbol{x}$ and denote $\boldsymbol{x}^*(\underline{h})$ the associated optimal solution.

3° Compute $h(\boldsymbol{x}^*(\underline{h}))$ according to (10).

4° If $\left| h - h(\boldsymbol{x}^*(\underline{h})) \right| < \varepsilon$ then stop else set $\underline{h} = h(\boldsymbol{x}^*(\underline{h}))$ and go to step 3.

As it can be noticed in ***Fig. 5*** or derived from the expressions (9), the direct fuzzy approaches make use only of the left hand side of the membership function. It means that a part of fuzzy number from F^2 to F^3 is not taken into account (see ***Fig. 5***).

To overcome this weakness of the above mentioned fuzzy approaches, there is another fuzzy approach [1], which makes use of the membership function on its whole range. This approach resembles way in which random coefficients are processed, when their distribution of probability is known. In this probabilistic-like approach the interval [0,1] of possible values of the membership function is divided by real numbers of an arbitrary chosen finite set $H \subset [0,1]$. Then for each fuzzy coefficient $\boldsymbol{c}$ from the location model the values $c^1, c^2, \ldots, c^r$ are determined, so that the constraint $\mu_c(c_k) \in H$ holds for $k=1,2,\ldots,r$. This is possible concerning fact that the level of satisfaction of a fuzzy number centre is 1.

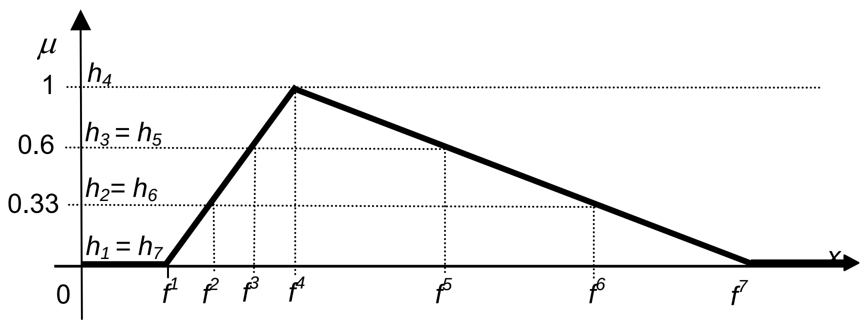

Fig.6 *Level of satisfaction assignment to values of membership function*

Then we minimize the weighted sum function over the feasible solutions $\boldsymbol{D}$.

$$min\left(\frac{\sum_{k=1}^{m+1}\sum_{i \in I} h_k f_i^k y_i}{\sum_{k=1}^{m+1} h_k} + \frac{\sum_{k=1}^{m+1}\sum_{i \in I}\sum_{j \in J} h_k c_{ij}^k z_{ij}}{\sum_{k=1}^{m+1} h_k} \right) \quad \text{subject to} \quad (\boldsymbol{y}, \boldsymbol{z}) \in \boldsymbol{D} \qquad (11)$$

The operating name of this method is **weights2**.

In the case, we don't have more accurate information about uncertain costs, it means $h_k = 1$ for $\forall k = 1,2,\ldots,m+1$, the method is named **minimum2**.

If we use results of classical sensitivity analyses $(\boldsymbol{y}, \boldsymbol{z}) \in \left\{ (\boldsymbol{y}, \boldsymbol{z})^1, (\boldsymbol{y}, \boldsymbol{z})^2, \ldots, (\boldsymbol{y}, \boldsymbol{z})^{m+1} \right\}$ in the weighted sum function and find for which of those results is its value minimal, it is the method **minimum1**. When we have nonzero weights, the method is named **weights1**.

To compare and verify both approaches, there were implemented [1] branch and bound method and built a software tool for sensitivity analysis and fuzzy processing of the location problem.

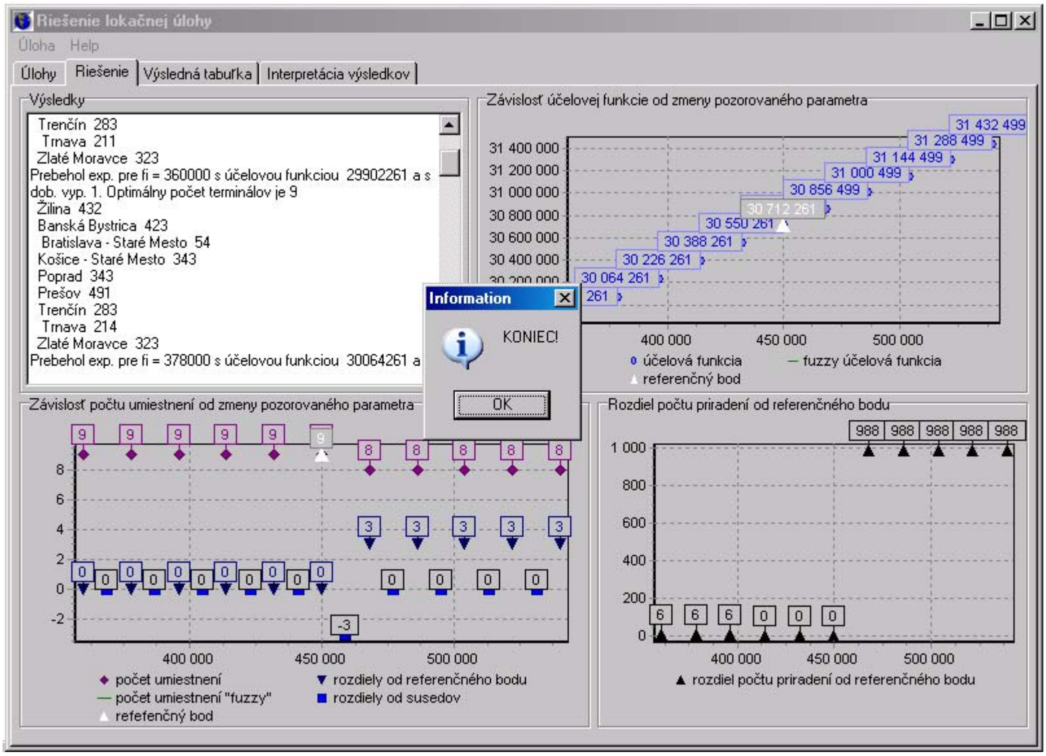

Fig. 7 *Graphical output of the program*

It is possible to change the values of choosen parameter and also the method – analysis of sensitivity, classical fuzzy approach or fuzzy algorithm. The results are: object time, value of objective function, optimal number of terminals and their names and also associated customers.

Functionality of the program was tested on 90 examples making use of the whole road system of Slovakia with 2906 dwelling places and 71 possible terminal locations. This way, in accordance with the primary source selection at 10 big towns of Slovakia 10 basic problems with predefined parameters f, e_1, e_0 were obtained. By three types of modification done independently with each of the three parameters, there 90 benchmarks were obtained, which were used in the experiments.

An average locations number for the method **weights2** is $\mathbf{9 \pm 0.9}$.

method	**difference of average locations number from 9±0.9**
weights1	$0.01 \Rightarrow 0.12$ %
classical fuzzy method	$0.49 \Rightarrow 5.39$ %
fuzzy algorithm	$1.09 \Rightarrow 12$ %
minisum1	$0.06 \Rightarrow 0.61$ %
minisum2	$0.06 \Rightarrow 0.61$ %

Tab. 1 *Differences of average locations number between weights2 and other methods*

Average objective function value for method ***weights2*** is $32294726 + 3190392\,Sk$.

method	**differences of average objective function value from** $32294726 + 3190392\,Sk$
weights1	*174298* $\Rightarrow$ *0.54 %*
classical fuzzy method	*660291* $\Rightarrow$ *2.04 %*
fuzzy algorithm	*1619143* $\Rightarrow$ *5.01 %*
minisum1	*172483* $\Rightarrow$ *0.53 %*
minisum2	*99119* $\Rightarrow$ *0.31 %*

Tab. 2 *Differences of average value of objective function between weights2 and other methods*

5. Conclusion

The fuzzy algorithm computes unique solution, which is not dependent on expert's ability (like classical fuzzy method) and is resistant to future changes.

We have compared these approaches:

- *sensitivity analysis* and its usage by methods *minisum1, minisum2, weights1, weights2,*
- *classical fuzzy method,*
- *fuzzy algorithm*.

If the uncertain costs of location problem are described by a triangular fuzzy number, both methods *weights2* and *fuzzy algorithm* are correct ways of finding the design of distribution system. These methods give similar results (see tables ***Tab 1*** and

Tab 2). There is difference 12% in number of placed terminals (average is 9 terminals). We suggest to perform both approaches and resulting design take into account only if results of these methods differ slightly. In opposite case, we suggest to perform an additional cost analysis and make the fuzzy cost more precise

Comparing of methods *minisum1* and *minisum2* with *weights2* is only the reference example, because these approaches don't take into account weights.

One of the program outputs is graphical representation of the solution, so user can find out the stability of the optimal solution (see ***Fig. 7***).

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Resumé

OPTIMÁLNÍ UMÍSTNĚNÍ DITRIBUČNÍCH CENTER PŘI NEJISTÝCH NÁKLADECH

Zuzana ŠVECOVÁ, Vladimír KLAPITA

V úlohách strategického rozhodování je značně problematické určení hodnoty budoucích nákladů. Tato nepřesnost může způsobit, že vypočítané řešení může být vzhledem k současným nákladům neefektivní. V našem článku jsme se snažili najít přístup, který je schopen vypočítat jediné řešení lokačního problému při nejistých nákladech, které je odolné vůči budoucím změnám. V článku je uvedena analýza citlivosti spojená s exaktní metodou matematického programování a teorií fuzzy množin.

Summary

OPTIMAL LOCATION OF DISTRIBUTION CENTERS UNDER UNCERTAIN COSTS

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It is fairly questionable to estimate future costs in the problems of strategic decision. The uncertainty may cause that a resulting solution could be very inefficient considering current costs. In our report we try to find approach, which enables to determine unique solution of location problem under uncertain costs so that solution be resistant to future changes. We deal with sensitivity analysis and with the connection of an exact mathematical programming method and theory of fuzzy sets.

Zusammenfassung

OPTIMALE LOKATION DER DISTRIBUTIVE ZENTREN BEI FRAGLICHEN KOSTEN

Zuzana ŠVECOVÁ , Vladimír KLAPITA

In Aufgaben den stretegischen Entscheidungen ist ziemlich problematisch die Verte die angehende Kosten zu feststellen. Das bedeutet, dass der Lösungzustand wird ungünstig in Beziehung zu wirklichen Kosten. Wir suchen Zutritt, welche errechnet die eindeutige Lösung für das Lokationproblem bei fraglichen Kosten, meist resident entgegen die angehende Umsatzes. Wir befassen mich mit der Sensitivitätanalyse und mit der Methode, welche aneinander fügen mathematische Programmierung mit der Teorie der Fuzzy Mengen.