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EXPERIENCE WITH USE OF A TIME SAVING METHOD FOR WHEEL-RAIL FORCES CALCULATION

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Overview of methods for calculation of wheel-rail forces

The calculation of wheel-rail forces is a crucial factor in computer simulation of railway vehicles dynamic behaviour. In the simulation, the computation of wheel-rail forces is repeated many times. Therefore a short calculation time is very important.

There are various methods for the calculation of wheel-rail forces. The best known methods can be divided into four groups:

- exact theory by Kalker (programme CONTACT)
- simplified theory by Kalker (programme FASTSIM)
- look-up tables
- simplified formulae and saturation functions.

The exact theory by Kalker (computer programme CONTACT [1]) has not been used in the simulations because of its very long calculation time.

The simplified theory used in Kalker's programme FASTSIM [2] is about 1000 times faster than the exact theory, but the calculation time is relative long for use in complicated multi-body systems. FASTSIM is used in the railway vehicle simulation tools ADAMS, MEDYNA, SIMPACK, GENSYS, VOCO. The Benchmark results [9] give usually longer calculation time in comparison with other programmes.

Another possibility for computer simulations consists in the use of look-up tables with saved pre-calculated values (ADAMS, VAMPIRE) . Because of the limited data in the look-up table, there are differences to the exact theory as well. Large tables are more exact, but the searching in such large tables consumes calculation time.

Searching for faster methods some authors found approximations with simple saturation functions (e.g. [4]). The calculation time using these approximations is short, but there are significant differences to the exact theory. Simple approximations are often used as a fast and less exact alternative to standard method (MEDYNA, VAMPIRE, SIMPACK).

Disadvantages mentioned above can be avoided by using the proposed algorithm described below. In spite of the simplifications used, spin is considered. Because of the short calculation time, the creation of a look-up table is not necessary. In comparison to other approximation methods, a smaller difference between the calculated values and the exact theory can be achieved.

The paper deals with the theoretical background of the algorithm, comparison with FASTSIM and experience of use of this method.

A time saving method and its theoretical background

The proposed method assumes the ellipsoidal contact area with semiaxis *a*, *b* and normal stress distribution according to Hertz. The maximum value of tangential stress τ at any arbitrary point is

$$\tau_{\max} = f \, . \, \sigma \qquad (1)$$

f .... coefficient of friction,

σ .... normal stress.

The coefficient of friction *f* is assumed constant in the whole contact area.

The solution assumes a linear growth of the relative displacement between the bodies from the leading point (A) to the trailing point (C) on the edge of the contact area (Fig. 1). At first the contact surfaces of the bodies stick firmly together and the displacement of the bodies is the result of material creepage (area of adhesion). The tangential stress τ acts against the creep and its value grows linearly with the distance from the leading edge (the assumption is identical with Kalker's simplified theory). If τ in the adhesion area reaches its maximum value according to (1) a relative motion of the contact surfaces appears. This part of contact area is called the area of slip. The tangential stress acts against the slip according to (1).

The tangential force is determined as

$$F = \iint_{(U)} \tau \, dx \, dy \,, \tag{2}$$

U .... contact area.

For vectors in x and y directions we get

$$F_i = F \; \frac{s_i}{s} \qquad\qquad i = x, y \,, \tag{3}$$

$$s = \sqrt{s_x^2 + s_y^2} \;, \tag{4}$$

s_x, s_y .... slip in x and y directions.

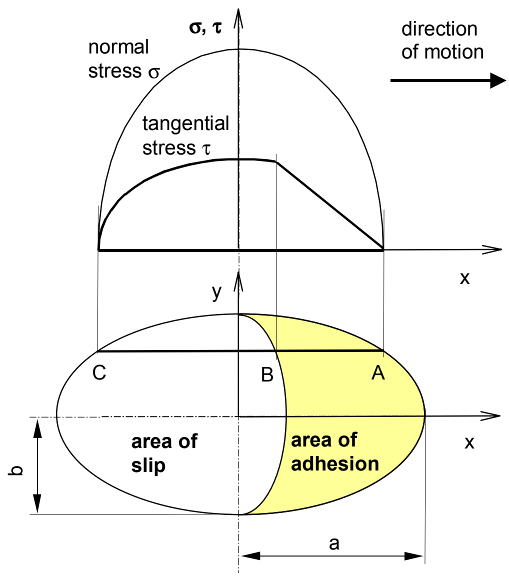

Fig.1 *Assumption of distribution of normal and tangential stresses in the wheel-rail contact area*

Freibauer [5] has solved the creep-force law without spin using a transformation of the tangential stress distribution ellipsoid to a hemisphere with the formulae

$$y^* = \frac{a}{b} \, y \,, \qquad \tau^* = \frac{a}{\tau_0} \, \tau \tag{5}$$

y^*, τ^* .... transformed variables y, τ

τ_0 .... the maximum stress in the centre of the contact area.

The tangential stress is proportional to slip s and the distance from the leading edge (Fig. 1) with proportionality constant *C*, which is a value characterising the contact elasticity of the bodies (tangential contact stiffness). The gradient of tangential stress in the area of adhesion is

$$\varepsilon = \frac{2}{3}\ \frac{C.\pi\,.\,a^2.\,b}{Q\,.\,f}\ s\,, \tag{6}$$

Q .... wheel load.

The tangential force is then

$$F = \tau_0 \frac{b}{a^2} \iint_{(U)} \tau^*\ dx\, dy^* = -\tau_0 \frac{b}{a^2} \frac{4}{3} a^3 \left(\frac{\varepsilon}{1+\varepsilon^2} + \arctan\varepsilon \right) \tag{7}.$$

According to the theory of Hertz

$$\tau_0 = \sigma_0\,.\,f = \frac{3}{2}\frac{Q\,.\,f}{\pi.\,a\,.\,b}\,, \tag{8}$$

σ_0 .... maximal normal stress in the contact area.

After substitution into (7) we obtain

$$F = -\frac{2.Q.\,f}{\pi} \left(\frac{\varepsilon}{1+\varepsilon^2} + \arctan\varepsilon \right) \tag{9}$$

The vector forces F_x, F_y are calculated from (9) using formulae (3).

When solving the wheel-rail contact problem, the spin is of a considerable importance. Spin is a rotation about the vertical axis z caused by wheel conicity. Further under the title spin we will understand the relative spin ψ which means the angular velocity about z-axis divided by speed *v*. The spin is

$$\psi = \frac{\omega.\sin\gamma}{v} = \frac{\sin\gamma}{r}\,, \tag{10}$$

ω .... angular velocity of wheel rolling

γ .... angle of contact surfaces

r .... wheel radius.

In the following part a force effect of the spin will be mentioned. The moment effect of spin as well as the moment effect caused by lateral slip are neglected because they are too small in comparison with other moments acting on the vehicle.

At a pure spin the vector force F_x is zero. The centre of rotation is situated on the longitudinal axis of the contact area, but its position depends on the equilibrium of the forces and is unknown at the beginning of the solution. If the longitudinal semiaxis is too small ($a \to 0$), the centre of spin rotation is approaching the origin of the co-ordinate system. Using the transformation of the tangential stress distribution ellipsoid to a hemisphere, the lateral tangential force caused by pure spin was found as

$$F_y = \tau_0 \frac{b}{a^2} \iint_{(U)} \tau_y^{\ *} \, dx \, dy^* = -\frac{3}{8} \pi \, . \tau_0 \, . a \, . b \left[|\varepsilon| \left(\frac{\delta^3}{3} - \frac{\delta^2}{2} + \frac{1}{6} \right) - \frac{1}{3} \sqrt{\left(1-\delta^2\right)^3} \right], \tag{11}$$

Using the formula (8) we get

$$F_y = -\frac{9}{16} \, Q . \, f . K_M \, , \tag{12}$$

$$K_M = |\varepsilon| \left(\frac{\delta^3}{3} - \frac{\delta^2}{2} + \frac{1}{6} \right) - \frac{1}{3} \sqrt{\left(1-\delta^2\right)^3} \, , \tag{13}$$

$$\delta = \frac{\varepsilon^2 - 1}{\varepsilon^2 + 1} , \tag{14}$$

and creep *s* in equation (6) is given as $\psi . a$. However this solution is valid only for $a \to 0$.

The detailed solution for different relations *a/b* given by Kalker [3] showed that with an increasing relation *a/b* the force effect of the spin grows. Looking for a fast solution to be used in simulations, a correction of dependence (11) for $a > 0$ was found. The forces caused by longitudinal and lateral creepages and the lateral force caused by spin are calculated separately. In the equations (3), (4) and (6) instead of the slip *s* there is resulting slip s_C

$$s_C = \sqrt{s_x^2 + s_{yC}^2} \, , \tag{15}$$

$$\begin{array}{lll} s_{yC} = s_y + \psi . a & for & \left| s_y + \psi . a \right| > \left| s_y \right| \\ s_{yC} = s_y & for & \left| s_y + \psi . a \right| \le \left| s_y \right| \end{array} . \tag{16}$$

The resulting force effect in lateral direction is given as the sum of both above described effects respecting the creep saturation as follows

$$F_{yC} = F_y + F_{yS} \, , \tag{17}$$

F_{yS} .... increase of the tangential force caused by the spin.

Its value is

$$F_{yS} = -\frac{9}{16}\ a\,.Q.\ f\,.K_M \left[1+6.3\left(1-e^{-\frac{a}{b}}\right)\right]\frac{\psi}{s_C}\,, \qquad (18)$$

where K_M is obtained by (13) and ε is actually given as

$$\varepsilon_S = \frac{2}{3}\ \frac{C.\pi\,.a^2.b}{Q\,.\,f}\ \frac{s_{yC}}{1+6.3\left(1-e^{-\frac{a}{b}}\right)}\,. \qquad (19)$$

The contact stiffness C in (6) and (19) can be found by experiments or can be obtained from Kalker's constants [3].

Using the proposed method as a fast substitution of Kalker's solution, the value of the tangential contact stiffness C is derived by assuming an identical linear part of the creep-force law according to Kalker theory and proposed method. According to the proposed theory, if the creep is close to 0 ($\varepsilon \to 0$), without spin

$$F = -\frac{8}{3}\,a^2.b\,.C\,.s\,, \qquad (20)$$

and according to Kalker

$$F = -\ G.a\,.b\,.c_{jj}\,.s\,, \qquad (21)$$

c_{jj} .... Kalker's constants (c_{11} for longitudinal direction, c_{22} for lateral direction).

After the comparison of (20) with (21) we get

$$C = \frac{3}{8}\frac{G}{a}\,c_{jj}\,, \qquad (22)$$

After the substitution of (22) in (6) the gradient ε of tangential stress is

$$\varepsilon = \frac{1}{4}\ \frac{G.\pi\,.a.b.c_{jj}}{Q\,.\,f}\,s\,. \qquad (23)$$

Because $c_{11} \neq c_{22}$, constant c_{jj} will be obtained as follows

$$c_{jj} = \sqrt{\left(c_{11}\frac{s_x}{s}\right)^2 + \left(c_{22}\frac{s_y}{s}\right)^2}\,. \qquad (24)$$

The lateral force caused by the spin (12) is for $\varepsilon \to 0$ according to the proposed theory

$$F_y = -\frac{1}{4}\pi . a^3 . b . C_S . \psi , \quad (25)$$

and according to Kalker's theory

$$F_y = -G . c_{23} . \psi . \sqrt{(a.b)^3} . \quad (26)$$

With comparison of (25) and (26) we get

$$C_S = \frac{4}{\pi} \frac{G.\sqrt{b}}{\sqrt{a^3}} c_{23} . \quad (27)$$

After the substitution of (27) in (19) the gradient of tangential stress ε_S used for calculation of spin influence is

$$\varepsilon_S = \frac{8}{3} \frac{G.b.\sqrt{a.b}}{Q.f} \frac{c_{23} . s_{yC}}{1 + 6.3\left(1 - e^{-\frac{a}{b}}\right)} . \quad (28)$$

Comparison with programme FASTSIM

The proposed method was compared with Kalker's programme FASTSIM. The comparison has been done in the non-dimensional co-ordinates

non-dimensional wheel-rail forces

$$f_i = \frac{F_i}{Q.f} \qquad i = x, y , \quad (29)$$

non-dimensional creep

$$\vartheta_i = \frac{G.a.b.c_{jj} . s_i}{Q.f} \qquad i = x, y; \quad j = 1 \text{ for } i = x, \ j = 2 \text{ for } i = y , \quad (30)$$

non-dimensional spin

$$\psi_n = \frac{G.\left(\sqrt{a.b}\right)^3 . c_{23} . \psi}{Q.f} . \quad (31)$$

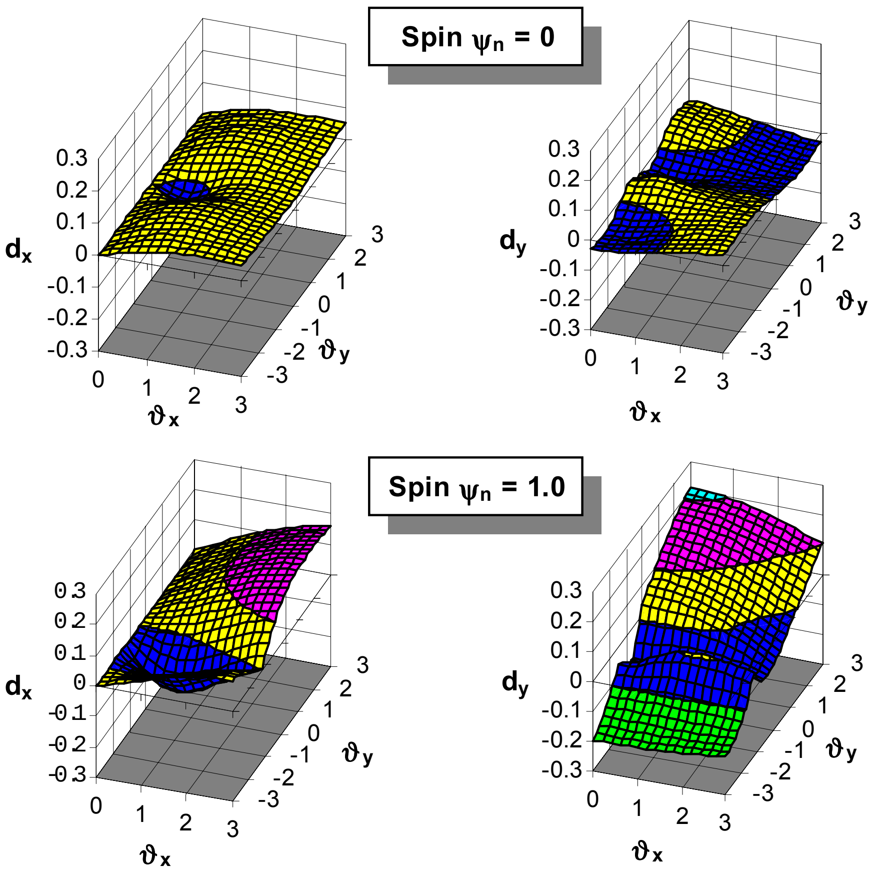

Fig. 2 *The difference between the two methods of calculation of the non-dimensional longitudinal (left) and lateral (right) forces in function of creep and spin*

Fig. 2 shows the difference of longitudinal and lateral non-dimensional forces under different creep and spin conditions computed with the proposed method and with the programme FASTSIM. The difference was defined as follows

$$d_i = f_{iK} - f_{iP} \qquad i = x, y , \qquad (32)$$

f_{iK} .... non-dimensional force computed with FASTSIM (*n*=30, where *n* is number of slices in FASTSIM)

f_{iP} .... non- dimensional force computed with the proposed method.

The greatest seldom occurred differences amount up to 0.3, which is acceptable for practical use in railway vehicle dynamics. However the proposed algorithm is about 17 times faster than the programme FASTSIM with recommended number of slices *n*=10 (Fig. 3). The computation time profit using this method in the simulations of rail vehicle dynamics depends, of course, on the structure of MBS-tools. Tests in different programmes give a 3 to 8 times faster calculation time as compared to FASTSIM.

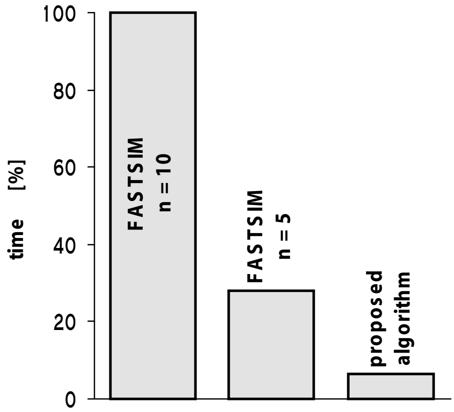

Fig. 3 *Comparison of calculation time (n – number of slices in FASTSIM)*

The FORTRAN computer code ADH containing the proposed algorithm was published in [8] together with some examples which allow to control the results and to apply this method in the user's own computer simulation programmes.

Comparison of simulation results using FASTSIM and using the proposed method

To compare the simulation results using the proposed method and FASTSIM, a lot of analyses have been done with an ADAMS/Rail model of Locomotive 2000 (SBB 460 of Swiss Railways), Fig. 4 and 5.

The model consists of a car body, two bogie frames, four wheelsets, four traction motors, rods of the wheelset steering system, two traction rods and springs and dampers of the primary and secondary suspensions. Altogether, the model contains

- 51 rigid bodies
- 84 bushing elements
- 4 bump-stops
- 24 dampers
- and possesses 266 degrees of freedom.

Fig. 4 *Locomotive SBB 460 of Swiss Federal Railways and its ADAMS/Rail model*

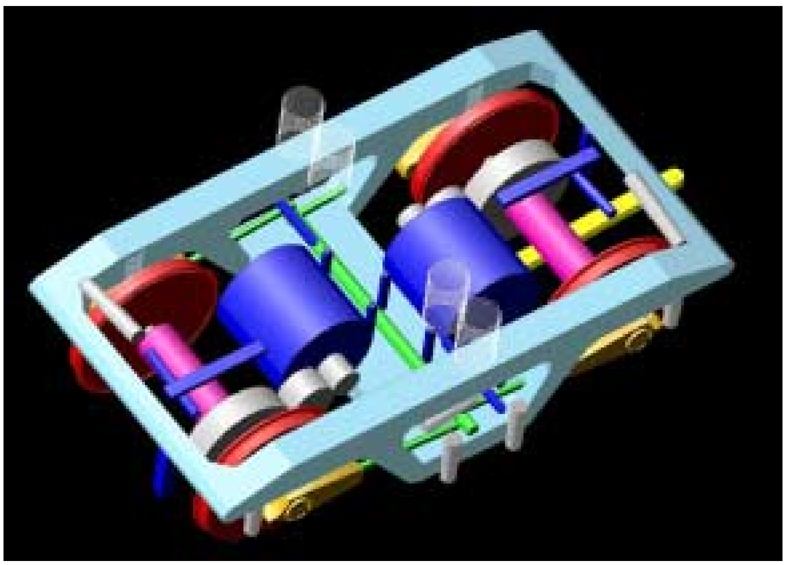

Fig. 5 *Model of bogie including drive system*

To make a comparison between the proposed method and FASTSIM, different curving analysis have been done with two models of Locomotive 2000. Fig. 6 shows the lateral forces and angles of attack for two variants of axle guidance stiffness. The results using both methods mentioned are very similar, however there is a big difference in the calculation time.

Of course, for practical use of the tools the verification by measurements is of great importance. The two investigated methods have been compared with measurements in a curve of 300 m radius. The measurements have been done during the type test of the locomotive. The lateral and vertical wheel-rail forces were measured with sensors fixed on the rail. The comparison is shown in Fig. 7 for low speed and lateral acceleration of -0.3 m/s^2 and for high speed and the lateral acceleration of 1.1 m/s^2. The comparison confirms that the results calculated using the proposed method show good agreement with the measurements. Specially at the leading wheelsets they are nearer to the actually measured values than the results obtained in the simulations using FASTSIM.

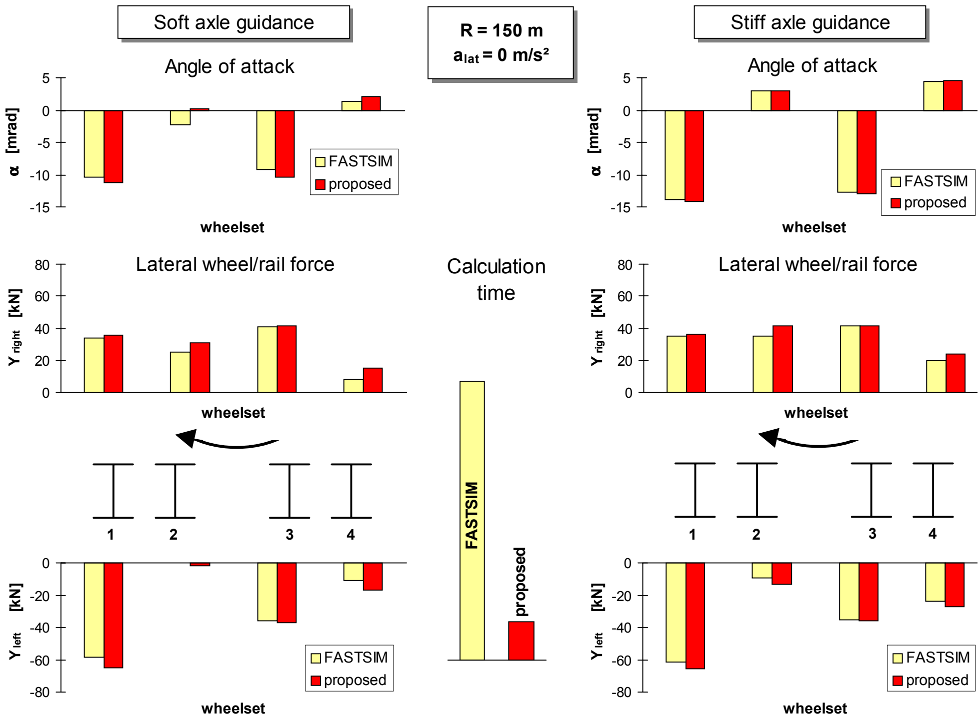

__Fig. 6__ Comparison of curving simulation results using FASTSIM and the proposed method (simulation package ADAMS/Rail)

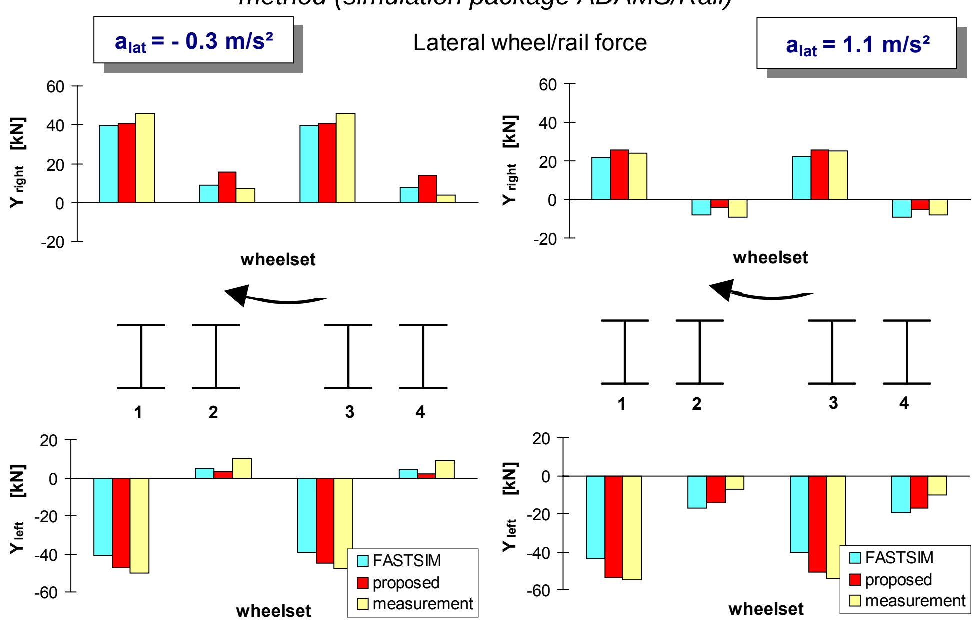

__Fig. 7__ Measured lateral wheel-rail forces in a curve with 300 m radius compared with simulations using the proposed method and using FASTSIM

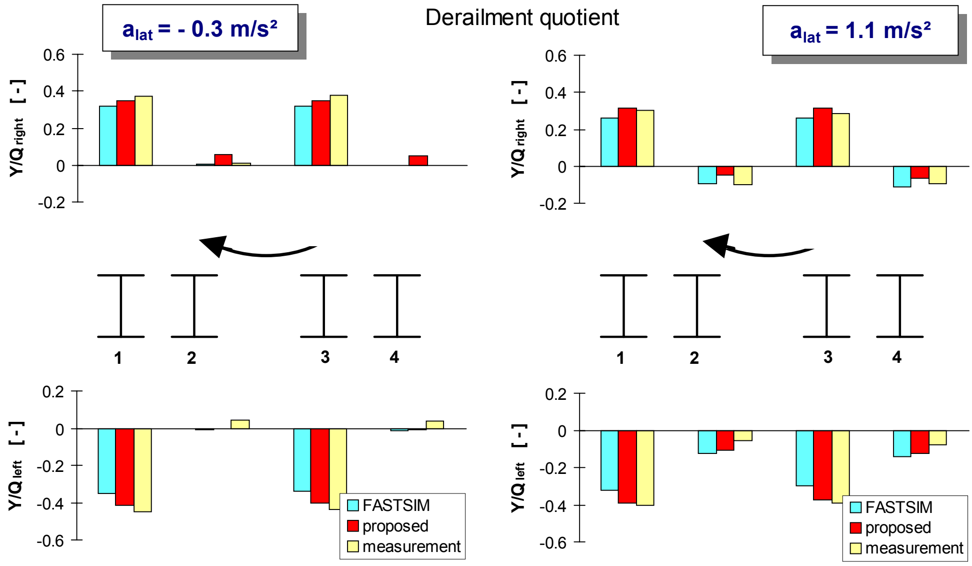

Fig. 8 Measured derailment quotient Y/Q compared with simulations using the two methods mentioned

Fig. 8 presents the comparison of derailment quotient Y/Q. The diagrams confirm good agreement with the measurements, specially at the simulations using the proposed method.

Experience with use of the proposed method in simulation tools

The proposed method has been used in the simulations in different programmes since 1990, e.g. [10] . In-between, positive experience has been achieved in the research as well as in the industrial application. The algorithm is used in ADAMS/Rail [7] as an alternative parallel to FASTSIM and to the table-book. The calculation time is faster than FASTSIM's and usually faster than the table-book as well. An analytic interpolation used in the proposed algorithm provides a smoothing of the contact forces in comparison with the table-book and there are no convergence problems during the integration. The programme was tested as a user routine in programmes: SIMPACK, MEDYNA, SIMFA and various user's own programmes with very good experience.

Conclusions

The paper presents the theoretical background and experience with use of a time saving method suitable for wheel-rail forces calculation in the simulation tools. The proposed algorithm

- allows the calculation of full non-linear wheel-rail forces
- takes spin into account
- can be used as a fast alternative to Kalker's method to save calculation time

- makes saving of pre-calculated values superfluous
- shows good agreement in comparison with measurements
- has been used in different simulation tools since 1990 with very good experience.

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Resumé

ZKUŠENOSTI S UŽÍVÁNÍM ČASOVĚ ÚSPORNÉ METODY VÝPOČTU SIL KOLO-KOLEJNICE

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Výpočet sil mezi kolem a kolejnicí se při simulaci jízdy kolejových vozidel na počítači mnohokrát opakuje, a proto je krátká výpočetní doba velmi důležitá. Příspěvek se zabývá metodou, která umožňuje značné zkrácení výpočetních dob. Navrhovanou metodu lze použít buď místo jednoduchých vzorců k dosažení přesnějších výsledků, nebo jako náhradu za Kalkerův program FASTSIM ke zkrácení výpočetních dob. Uvedená metoda dosahuje dobré shody s měřením, je používána v různých simulačních programech od r. 1990 a velmi dobře se osvědčila.

Summary

EXPERIENCE WITH USE OF A TIME SAVING METHOD FOR WHEEL-RAIL FORCES CALCULATION

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In the simulations of rail vehicle dynamic behaviour, the computation of wheel-rail forces is repeated many times. Therefore a short calculation time is very important. A time saving method for the computation of wheel-rail forces is presented in the paper. The proposed method can be used instead of simple formulas to improve the accuracy or as a substitution of Kalker's programme FASTSIM to save the computation time. The method shows good agreement in comparison with measurements and has been used in different simulation tools since 1990 with very good experience.

Zusammenfassung

ERFAHRUNGEN MIT ANWENDUNG EINER ZEITSPARENDEN METHODE FÜR BERECHNUNG DER RAD-SCHIENE-KRÄFTE

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In der Computersimulation der Fahrt der Schienenfahrzeuge wird die Berechnung der Kräfte zwischen Rad und Schiene vielmal wiederholt. Eine kurze Rechenzeit ist deshalb sehr wichtig. Im Artikel wird eine zeitsparende Methode für die Berechnung der Rad-Schiene-Kräfte präsentiert. Diese Methode kann entweder statt der einfachen Formel verwendet werden, um die Genauigkeit zu verbessern, oder an Stelle des Programms FASTSIM von Kalker, um die Rechenzeit zu reduzieren. Die Methode erreicht eine gute Übereinstimmung mit Messungen und ist in Anwendung in verschiedenen Simulationsprogrammen seit 1990 mit sehr guten Erfahrungen.